

Income Insecurity and Youth Emancipation: A Theoretical Approach*

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Abstract

In this paper, we propose a theoretical model to study the effect of income insecurity of parents and offspring on the child's residential choice. Parents are partially altruistic toward their children and will provide financial help to an independent child when her income is low relative to the parents'. We find that children of more altruistic parents are more likely to become independent. However, first-order stochastic dominance (FOSD) shifts in the distribution of the child's future income (or her parents') have ambiguous effects on the child's residential choice. Parental altruism is the very source of ambiguity in the results. If parents are selfish or the joint income distribution of parents and child places no mass on the region where transfers are provided, a FOSD shift in the distribution of the child's (parents') future income will reduce (raise) the child's current income threshold for independence.

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JEL Classification: D1, D8, J1, J2.

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1 Introduction and Literature Review

The age at which children leave the parental home differs considerably across countries. In 2002, for men aged 25 to 29 years old, some of the lowest coresidence rates in the European Union (EU) could be found in France, the Netherlands and the UK, ranging from 20 to 22%. In Italy, by striking contrast, the coresidence rate for the same group was 73%. Other southern European countries shared the Italian record, such as Greece (70%), Spain (67%), and, to a lesser extent, Portugal (58%). Similar disparities were present among women.

Coresidence decisions have important implications for a variety of social phenomena. Fertility decisions are one such example, with obvious consequences for the sustainability of social security programs.

Interestingly, the literature on the determinants of household membership is mostly empirical in nature.¹ McElroy (1985) examines the joint determination of labor supply participation and household membership in the United States. Also for the US, Rosenzweig and Wolpin (1993) study the properties of financial transfers from parents to their young adult sons, as well as transfers in the form of shared residence, with particular attention posed on the child's accumulation of human capital. Other contributions include Ermisch (1999), for the UK, and Card and Lemieux (2000), for Canada and the US. This body of literature focusses on the effects of the income of parents and children and of housing prices on the coresidence decisions of youth.²

In this paper, we propose a theoretical model to study the effect of income insecurity of parents and offspring on the child's residential choice. Specifically, in a dynamic environment where both the child and the parents' future incomes are uncertain, we examine whether shifts in the distribution of future incomes affect the child's coresidence

¹We discuss rare exceptions such as Ermisch (2003) and Fogli (2004) below.

²Regarding the southern European experience, Manacorda and Moretti (2006) have emphasized the income of parents in Italy (whom, they argue, bribe their children to stay at home), whereas housing costs were examined in Giannelli and Monfardini (2003), for Italy, in Martinez-Granado and Ruiz-Castillo (2002), for Spain, and in Martins and Villanueva (2006), for Portugal. Giuliano (2007) proposes a higher desirability of living at home due to the increased freedom for young adults brought forth by the "sexual revolution" of the late 1960s.

choice. While also having predictions for the effects of current income on coresidence, the novel element of our research relative to the household formation literature is the focus on income insecurity as a determinant of coresidence.

In addition to household formation, our analysis is also closely related to the literature on altruism, an important instance of interdependent utilities.³ Theoretical models in this literature generally are of the overlapping-generations type and take the formation of new households as *exogenous*. Our model assumes parents to be partially altruistic⁴ – parents care about their children although less so than about themselves – but the flexible parameterization we use allows us to consider the extreme cases of full or no altruism as well. In contrast to much of the altruism literature, we *endogenize* the child’s decision to form a new household and examine how this decision depends on the degree of altruism and on expectations of future income.

More specifically, in our model, parents are partially altruistic toward their children and will provide financial help to an independent child when her income is low relative to the parents’. However, if a child coresides with her parents, we assume she will have access to a greater share of total familial income than granted to her through financial transfers in the state of independence. This assumption is rooted on the difficulty of excluding the child from the consumption of public goods such as housing. Moving out is costly; in fact, in our setup, moving out is irreversible.⁵ We consider two dimensions of income insecurity, corresponding to shifts in the distribution of income in the sense of first- and second-order stochastic dominance (abbreviated FOSD and SOSD, respectively). While one well-known implication of FOSD shifts in the income distribution is for expected income to increase, under SOSD expected income is held constant and it is the variance of the income process that declines.

We show that FOSD shifts in the distribution of the child’s future income (or her

³See Barro (1974) and Becker (1974) for early and classic examples, and Laitner (1997) for an overview.

⁴Laitner (1988) is one instance of partial altruism.

⁵We will argue later that this assumption carries no loss of generality as compared to finite moving costs while providing substantial tractability gains.

parents') will have ambiguous effects on the child's residential choice. The reason is as follows. For income pairs such that parents provide transfers to their independent children, higher income (either the child's or her parents') raises the child's consumption both at home and when independent. Partially altruistic parents will only provide transfers to children whose consumption is lower than their own. Therefore, when transfers are provided, if the child or the parents' income increases, while consumption at the parental home goes up by more than consumption when independent, the marginal utility of an extra unit of consumption is highest for an independent child. Consequently, for the range of income values such that transfers are positive, the impact of the child's higher income (or her parents') affects the differential utility across the two residential states in an ambiguous way. Further, while some parameter values allow us to solve this ambiguity for one family member, we show that the ambiguous effect of income on coresidence cannot be simultaneously eliminated for both parent and child.

Our analysis identifies parental altruism as the very source of the ambiguous impact of higher income on the child's residential status. Absent altruism, since transfers will no longer be given to independent children, the intuitive results that FOSD shifts in the distribution of the child's (parents') future income reduce (raise) the child's current income threshold for independence do emerge. More generally, in the presence of altruism, these results only hold true when the joint income distribution of parent and child places no mass on the region where transfers are made. The altruism driven ambiguity of the impact of income changes on the child's residential status has implications for SOSD income shifts, as well. Once again, unambiguous results only emerge for either selfish parents or by confining attention to income distributions such that positive transfers do not take place. In these cases, SOSD shifts in the distribution of the child's (parents') income reduce (raise) the child's income threshold for independence. Although altruism is the source of ambiguity regarding changes in income expectations, children of more altruistic parents will be better off when independent for the range of incomes that triggers transfers. As such, for these children, moving-out is a better prospect than for those of less caring parents. In this sense, and holding other things constant, "love" will push

children toward independence.

Other than our work, Fogli (2004) is the only other reference we are aware of that explicitly considers expectations of future income as a determinant of household membership decisions.⁶ While sharing the common concern of the effects of income uncertainty on coresidence, our analysis and goals are very different. Fogli starts from the interesting realization that countries with tight credit constraints also display high coresidence rates and high degrees of employment protection. She then argues that, given the credit constraints, employment protection for the parents is the outcome of a bargaining process between the young and old generations. Using an overlapping generations model, she studies the political economy environment of her economy under general equilibrium, focusing on whether or not the institutional environment of real economies may be an optimal outcome in the context of her model. Ours is a partial equilibrium model that analyzes the residential choice of one person at a time, and considers this individual in its relations with her family members.

Ermisch (2003) proposes a theoretical model of coresidence. Utility is defined over consumption and housing. He studies the effects of changes in the current income of parent and child as well as of changes in housing price on coresidence. In our model, utility is derived from consumption alone, expressed as the difference between income and housing costs. We do not consider housing as an independent argument since we find it unlikely that parents would adjust their living arrangements primarily in response to changes in the residential status of their child.⁷ Further, we consider a specific functional form for preferences. Although less general in these dimensions, our model allows us to shed light on the complex problem of the impact of income expectations on current residential choices.

⁶In Fogli, the variability of future income depends only on the probability of job loss as a worker ages. We consider general shifts in the distribution of future income.

⁷Even if the parents were able to reduce their own housing costs by switching to a smaller house upon the child moving out, economies of scale in housing imply that the total residential costs of the family increase with independence. This is all we need for our results to go through unchanged. On a related front, Ermisch (1999) documents a low housing price elasticity for the UK, suggesting that price effects are also negligible.

In the next section, we present our model and results. Section 4 concludes.

2 A model of job insecurity and coresidence

In this section we illustrate how coresidence decisions are related to job insecurity of parents and children using a dynamic, two-period model of residential choice. All proofs can be found in the Appendix.

2.1 The family

The family in our model has n_0 parents and $n_1 + 1$ children. We assume that it has either one or two parents, and at least one child. Family size is denoted n (with $n = n_0 + n_1 + 1$). Our focus is on the residential choice of one of the children, assuming that her siblings remain with the parents.

Direct utility is defined over consumption only.⁸ We assume that, in the parental home, all individuals pool income and consume an equal fraction of total familial income. If all family members are coresiding, then consumption in the parental home is given by:

$$c_p^n = \frac{y_p + y_c - \gamma_p}{n}, \quad (1)$$

where γ_p is the rent or the imputed cost of housing, y_p parental income, and y_c the income of the child who is contemplating to move out (her siblings are assumed to earn no income).

We rationalize this sharing rule as follows. At the parental home, parents choose the “lifestyle:” they choose the type of house the family lives in, the size of the child’s room, the car and the meals that the family enjoys. The child consumes these – partially public – goods that were not chosen by her (at least not completely). The “sharing rule” in (1) thus reflects the difficulty of excluding the child from consuming the parental lifestyle. Further, because the enjoyment of income is conditioned by the lifestyle choice (e.g. the child cannot enlarge her room to get more space even if her income increases), the sharing

⁸Utility from privacy is addressed at the end of section 2.3.

rule also implies that an additional unit of the child's income will materialize into greater consumption in the state of independence (see below): just as the parents cannot exclude the child from enjoying an increase in their income, the child cannot exclude other family members from also enjoying the fraction $(n - 1) / n$ of hers. When independent, she is free to choose her own lifestyle. Because of the presence of public goods in the household and of the partial rigidity associated with consumption patterns once, say, housing size and space distribution are fixed, we really think of the sharing rule as a *technology for sharing income* in the household.⁹ Our results would generalize to other sharing rules provided they were monotonic in income of all family members and the child got a higher fraction of familial resources when at home relative to independence.

We denote the child's consumption by c_c . If she stays, she gets c_p^n . If she moves out, she will consume all of her income net of housing costs under independence, γ_c , plus a non-negative transfer t from her parents:

$$c_i = y_c + t - \gamma_c.$$

Per capita consumption of the family members of an independent child is:

$$c_p^i = \frac{y_p - t - \gamma_p}{(n - 1)}.$$

The child's residential decision affects the way resources are divided in the family. By moving out, there is one fewer person with whom to divide income in the parental

⁹We have also chosen not to pursue a Nash bargaining framework for the following reasons. A dynamic (two-period) bargaining problem would force first period consumption allocations to depend on whether independence favors or hurts parents and child in terms of the corresponding expected utility in the second period, and on how the income distribution affects future residential states. For example, parents would give the child a larger first-period share of resources under coresidence if their expected future utility were higher under coresidence. Presumably, if the child's income distribution shifted in a way favorable to her future independence, they would do so as well. In turn, this intertemporal interdependence would greatly challenge the characterization of the consumption allocations and render intractable the analysis of how the child's utility differential across residential states varies with shifts in the distribution of income. The sharing rule in (1) does not face these hurdles because it stays constant over time. In addition, underlying our choice of sharing rule and its interpretation as a technology for sharing income, there is a feasibility aspect that limits the applicability of bargaining. Suppose parents and child want to give the child a greater share of familial consumption. Full enjoyment of higher income may require tearing down some walls to enlarge the child's room. The costs of such an operation would likely outweigh the bargaining surplus that was to be explored.

home, and there is also less income to share; further, an independent child may receive a transfer from her parents. The child's choice to become independent therefore also modifies consumption of those who stay home.

In our model, parents are partially altruistic. They weigh their direct utility by a factor $\lambda \in (0.5, 1)$, and their children's utility by only $(1 - \lambda)$. Parental utility is then:

$$U_p = \lambda \left(n_0 + \frac{(1 - \lambda)}{\lambda} n_1 \right) u(c_p) + (1 - \lambda) u(c_c). \quad (2)$$

In what follows, we will in fact use the slightly modified functional form:

$$U_p = \lambda (n_0 + n_1) u(c_p) + (1 - \lambda) u(c_c) = \lambda (n - 1) u(c_p) + (1 - \lambda) u(c_c), \quad (3)$$

which puts more weight on the utility of the n_1 children who always remain at home and simplifies the algebra significantly, while leaving our results qualitatively unchanged.

To obtain sharper results, we conduct our analysis using Constant Relative Risk Aversion (CRRA) for the direct utility from consumption: $u(c) = (1 - \alpha)^{-1} c^{1 - \alpha}$, with $\alpha > 0$.¹⁰

2.2 Timing

There are two periods, 1 and 2, with time corresponding to the second subindex of each income variable. In period 1, parent and child observe their income realizations, y_{p1} and y_{c1} . To ensure nonnegative consumption, we assume there is a lower bound on income realizations given by the housing costs, γ_p and γ_c . A positive income realization for the parent, interpreted as a draw of $y_{p1} > \gamma_p$, is equivalent to a job offer, and similarly for the child. Since there is no disutility from work, job offers are always accepted.¹¹ The child then decides whether or not to move out and parents subsequently choose transfers. Finally, consumption takes place as a function of the residential choice of the child, income realizations and parental transfers.

¹⁰The results generalize to other commonly used families of functions (such as the Constant Absolute Risk Aversion case).

¹¹Family member j would require a positive income threshold above γ_j before accepting a job offer if there were disutility from work or if individuals were productive while unemployed (through home production, say). We are ignoring these cases.

The main difference across periods comes from assuming that moving out is irreversible. This can be justified on the grounds that the direct costs from moving, as well as the social stigma attached to going back to the parental house, tend to make independence a rather permanent state. While qualitatively similar results would emerge from considering finite costs instead, irreversibility is of great analytical convenience. For a child who stayed with her parents in period 1, the period 2 timing of events and choices repeats itself. If the child has moved out in period 1, however, she faces no residential choice in period 2.¹²

2.3 Period 2

We now characterize the resource allocation and residential decision in period 2. Assuming that the incomes of parent and child have taken the values y_{p2} and y_{c2} , the optimal transfer the parent would give the child if she decided to move out solves the following problem:

$$\max_{t_2 \geq 0} \left\{ \lambda(n-1) u \left(\frac{y_{p2} - t_2 - \gamma_p}{n-1} \right) + (1-\lambda) u(y_{c2} + t_2 - \gamma_c) \right\}. \quad (4)$$

First-order conditions yield:

$$\lambda u'(c_{p2}^i) \geq (1-\lambda) u'(c_{i2}), \quad (5)$$

holding with equality when $t_2 > 0$. Since $\lambda > 0.5$, this implies that a transfer-receiving child has lower consumption than the remaining family members. If she has not moved out in period 1, a child whose income is low enough to trigger transfers will therefore prefer not to move out. For such a child, consumption at home will be higher for two reasons. At home she gets the higher fraction $1/n$ of total familial income compared to a smaller fraction when independent.¹³ In fact, the sharing rule in place at the parental home, where each individual gets the fraction $1/n$ of total income net of rent,

¹²Framed in game theoretical terms, our subsequent analysis studies the set of pure-strategy, subgame-perfect equilibria of this sequential game between parent and child.

¹³When independent, she gets the fraction $(\Gamma(n-1)+1)^{-1}$ of total familial income, with $\Gamma = (\lambda/(1-\lambda))^{\frac{1}{\alpha}} > 1$. It can be shown that $1/n > (\Gamma(n-1)+1)^{-1}$.

corresponds to the case of full altruism ($\lambda = 0.5$). By staying home, children are able to secure consumption of certain goods since parents cannot limit the child's consumption of those goods; when the child leaves, on the other hand, parental transfers represent fully voluntary payments to the child and, as such, reflect the partial nature of altruism. The second reason why the child's consumption will be higher if she stays home is the fact that, by doing so, the family's aggregate resources net of housing costs are higher as only one rental payment is made.¹⁴

We now address the moving out decision for the child who decided to stay at home in period 1. Define Δ_2 as the excess utility level when independent relative to coresiding, for period 2:

$$\Delta_2(y_{c2}, y_{p2}) \equiv u(c_{i2}) - u(c_{p2}^n).$$

Δ_2 is a function of the income realizations of parent and child in the current period.¹⁵ The child moves out if $\Delta_2 > 0$. If indifferent, $\Delta_2 = 0$, we assume she stays. Understanding the child's residential choice and how it is affected by changes in y_{p2} and y_{c2} crucially hinges on the properties of this function. We first address how different values of y_{c2} impact the child's residential choice and later address the effects of parental income.

How does Δ_2 change as a function of y_{c2} ? To answer this question, it is important to define two income thresholds. Define \tilde{y}_{c2} as the value such that parental transfers are zero, $t_2(\tilde{y}_{c2}) = 0$,¹⁶ and let \bar{y}_{c2} be the income value that makes the child indifferent between staying at the parental home or moving out, $\Delta_2(\bar{y}_{c2}) = 0$. Under CRRA preferences,

$$\tilde{y}_{c2} = \frac{y_{p2} - \gamma_p}{\Gamma(n-1)} + \gamma_c, \quad \bar{y}_{c2} = \frac{y_{p2} - \gamma_p}{n-1} + \frac{n}{n-1}\gamma_c, \quad (6)$$

with $\Gamma = (\lambda/(1-\lambda))^{\frac{1}{\alpha}} > 1$. It is easy to see that \bar{y}_{c2} exceeds \tilde{y}_{c2} .

¹⁴The analysis would not be modified if we allowed the child's income to be any nonnegative amount, as follows. Say that the parent's income is always enough to make both rental payments: $y_p \geq \gamma_p + \gamma_c$. Then, even if an independent child cannot afford her rent, the altruistic parent will still willingly make a transfer to her high enough so as to ensure that she receives positive consumption. That this is so can be inferred from the first-order condition for transfers (5) and the fact that the utility function we are using satisfies Inada conditions.

¹⁵As such, $\Delta_2(\cdot)$ is defined over $[\gamma_c, \infty) \times [\gamma_p, \infty)$.

¹⁶The notation $t_j(x)$ omits, for simplicity, other arguments of the function $t_j(\cdot)$. Similar simplifications will be used for $\Delta_2(\cdot)$ and other functions, throughout.

Lemma 1 below characterizes formally how Δ_2 depends on the child's income.¹⁷

Lemma 1 (Utility differential and the child's income) *The function $\Delta_2(y_{c2})$ is strictly negative for $y_{c2} \in [\gamma_c, \bar{y}_{c2})$ and strictly positive for $y_{c2} > \bar{y}_{c2}$. Further, $\Delta_2(y_{c2})$ is strictly increasing in the range $(\tilde{y}_{c2}, \bar{y}_{c2})$. When the relative-risk aversion parameter α exceeds 1, $\Delta_2(y_{c2})$ is strictly increasing for $y_{c2} \in (\gamma_c, \tilde{y}_{c2})$. When α is below 1, $\Delta_2(y_{c2})$ is strictly increasing for $y_{c2} > \bar{y}_{c2}$.*

Figure 1A depicts a possible configuration of $\Delta_2(y_{c2})$. As Lemma 1 shows, utility from independence exceeds that under coresidence for $y_{c2} \geq \bar{y}_{c2}$. Thus, a child who did not move out in period 1, will now leave if her income exceeds \bar{y}_{c2} ; otherwise she will stay. The set of income values $[\gamma_c, \bar{y}_{c2}]$ is a *regret region*: the child would prefer to go back home. As explained above, the sources of regret are the rental cost γ_c and partial altruism.

The dependence of the residential choice on y_{p2} is also of interest. Define \tilde{y}_{p2} as the level of parental income such that $t_2(\tilde{y}_{p2}) = 0$,

$$\tilde{y}_{p2} = (y_{c2} - \gamma_c) \Gamma(n - 1) + \gamma_p.$$

We note that \tilde{y}_{p2} is always well defined (that is, \tilde{y}_{p2} always exceeds γ_p for any value of y_{c2}). Let \bar{y}_{p2} denote the parental income level that leaves the child indifferent between moving out and coresiding, $\Delta_2(\bar{y}_{p2}) = 0$.

$$\bar{y}_{p2} = y_{c2}(n - 1) - n\gamma_c + \gamma_p.$$

For very low values of the child's income, \bar{y}_{p2} is not well-defined (that is, $\bar{y}_{p2} \leq \gamma_p$ whenever $y_{c2} \leq (n/(n - 1))\gamma_c$). In these cases, no parental income value will make independence preferable to coresidence. Define \hat{y}_{p2} as:

$$\hat{y}_{p2} = \max\{\gamma_p, \bar{y}_{p2}\},$$

and it follows that $\gamma_p \leq \hat{y}_{p2} \leq \tilde{y}_{p2}$ for all values of y_{c2} . Then,

¹⁷The function $\Delta_2(\cdot)$ will have kink points at \tilde{y}_{c2} and \tilde{y}_{p2} , the latter defined below. As such, it is not differentiable everywhere in its domain. However, one-sided derivatives are always well-defined. In what follows and in the proofs of the Appendix, at kinks, the appropriate side derivative will be considered.

Lemma 2 (Utility differential and the parent’s income) *The function $\Delta_2(y_{p2})$ is strictly decreasing for $y_{p2} \in [\gamma_p, \tilde{y}_{p2})$ and strictly negative for $y_{p2} > \hat{y}_{p2}$. For $y_{p2} \geq \tilde{y}_{p2}$, when the relative-risk aversion parameter α exceeds unity, $\Delta_2(y_{p2})$ is strictly increasing.*

In Figure 1B we depict a possible configuration for $\Delta_2(y_{p2})$. Whether or not $\Delta_2(\gamma_p)$ is positive depends on parameter values (specifically, a large number of family members n and a small rental cost γ_c make $\Delta_2(\gamma_p)$ positive). As Lemma 2 shows, however, for $y_{p2} > \hat{y}_{p2}$, $\Delta_2(y_{p2}) < 0$ holds unambiguously, and children of wealthy parents who stayed home will not move out. Just as with $\Delta_2(y_{c2})$, higher parental income does not necessarily raise the child’s willingness to stay home.¹⁸

The potential lack of monotonicity of the function $\Delta_2(\cdot)$ and the general configuration of this function are of great relevance for our results. In section 2.4, we discuss these properties in substantive detail and provide some intuition for the general results of Lemmas 1 and 2.

In Figure 2, we plot the curves \tilde{y}_{c2} and \bar{y}_{c2} in (y_c, y_p) space. To the right of the $\bar{y}_{c2}(y_{p2})$ schedule, the child moves out; to the left she stays. From the point of view of the moving-out decision taken in period 1, we can also divide the coresidence area into two parts. To the left of $\tilde{y}_{c2}(y_{p2})$, children who became independent in period 1 will receive a transfer, i.e. $t_2(y_{c2}, y_{p2}) > 0$, while to the right they will not. Recall that the coresidence area is a regret area.

We now briefly discuss our treatment of privacy gains and saving. The analysis could easily accommodate a taste for privacy on the part of children (or the parents) as long as it were separable from the utility from consumption. In such a case, the utility from privacy would add to the utility from consumption, raising the value of $\Delta_2(\cdot)$, but not modifying its curvature properties characterized above. Provided privacy gains are not

¹⁸A generalization to a situation where the sharing rule depended on (y_c, y_p) and the rule stayed constant over time could easily be made as follows. Say that the child gets at least as high a fraction of total familial resources as when independent. This is reasonable since the parent is altruistic and would want to give the child that large a share of resources. Say that, as the child’s income increases, her share of income approaches the share under full altruism, $1/n$. Then, this would simply lead to an upward shift of the function $\Delta_2(\cdot)$, reducing expected regret (see below) and making the child more willing to leave home. However, the lack of monotonicity of $\Delta_2(\cdot)$ would still be present here. Most of all our results would go through unchanged, in this scenario, with one likely exception being Lemma 4.

the driving force for independence – *i.e.* provided the shift in $\Delta_2(\cdot)$ were small so that income sharing would remain the more important determinant of utility differentials across residential states – the analysis would remain largely unchanged.

We have thus far ignored the possibility of saving. In particular, it could be the case that both parent and child are saving for the downpayment on a house for the child. (This is a common occurrence in Southern Europe.) We could then think of y_p and y_c as income net of saving, the income that is allocated to consumption. The saved amount will never be consumed since it will be used for the downpayment if the child becomes independent, and mortgage costs are summarized in γ_c . Thus, the consumption flows described above – and the associated comparison of utility across residential states – would not be modified. If the child were severely liquidity constrained, so that purchasing an apartment or renting one were not feasible, this would correspond to a very high value of γ_c (or, equivalently, to a very low value of $y_{c2} - \gamma_c$). As shown above, if $y_{c2} - \gamma_c$ is very low, in particular when $y_{c2} \leq \bar{y}_{c2}$, the child would prefer to stay home. Therefore, the model can also accommodate saving for house acquisition and liquidity constraints.¹⁹

2.4 Period 1

A simplified presentation of the model's structure is given in Figure 3. In period 1 the residential choice is more involved than in period 2 due to irreversibility and the possibility of regret. Naturally, the latter depends on the likelihood that period 2 incomes fall to the left of the schedule $\bar{y}_{c2}(y_{p2})$, in the regret region. We assume that $(y_{c2}, y_{p2}) \sim F(y_{c2}, y_{p2})$, where $F(\cdot)$ is the joint cumulative distribution function (cdf) of period 2 income (y_{c2}, y_{p2}) , with marginal cdfs $F_c(y_{c2})$ and $F_p(y_{p2})$. $F(\cdot)$ has support over $[\gamma_c, \infty) \times [\gamma_p, \infty)$.

¹⁹For the sake of tractability, the analysis abstracts from strategic saving issues such as those in Buchanan's (1975) Samaritan's Dilemma.

3 Results

Let R denote the regret region.²⁰ If $F(\cdot)$ assigns positive probability to R , staying home in period 1 has an *option value*, the value associated with waiting to see the realization of the period 2 income and deciding then whether or not to move out. Just like with any real option, this value has to be weighted against the potential gains from moving out early on.

Define Δ_1 as the expected excess utility from moving out relative to staying home in period 1, conditional on making the optimal residential choice in period 2:

$$\Delta_1(y_{c1}, y_{p1}) \equiv u(c_{i1}) + \int_{\gamma_p} \int_{\gamma_c} u(c_{i2}) dF(y_{c2}, y_{p2}) - \left\{ u(c_{p1}^n) + \int_{\gamma_p} \left[\int_{\gamma_c}^{\bar{y}_{c2}(y_{p2})} u(c_{p2}^n) dF_c(y_{c2}|y_{p2}) + \int_{\bar{y}_{c2}(y_{p2})}^{\infty} u(c_{i2}) dF_c(y_{c2}|y_{p2}) \right] dF_p(y_{p2}) \right\}.$$

Δ_1 is defined over the period one incomes of parent and child.²¹

The first two terms in Δ_1 represent the expected utility from moving out in period 1. Given that the child becomes independent in period 1, period 2 utility is also computed for $c_{c2} = c_{i2}$. The terms preceded by a minus sign represent the expected utility from staying home in period 1. In this case, the child retains the possibility of choosing the best residential arrangement in period 2. Thus, given y_{p2} , for $y_{c2} \leq \bar{y}_{c2}(y_{p2})$, the child remains with her parents and $c_{c2} = c_{p2}^n$; otherwise she moves out and $c_{c2} = c_{i2}$. The child will move out if $\Delta_1 > 0$. When $y_{c2} > \bar{y}_{c2}(y_{p2})$, having moved out in period 1 does not carry any utility loss; therefore, in this range, the terms concerning period 2 utility while independent cancel out and the moving out condition $-\Delta_1 > 0$ simplifies to:

$$u(c_{i1}) - u(c_{p1}^n) > \int_{\gamma_p} \left[\int_{\gamma_c}^{\bar{y}_{c2}(y_{p2})} (u(c_{p2}^n) - u(c_{i2})) dF_c(y_{c2}|y_{p2}) \right] dF_p(y_{p2}). \quad (7)$$

²⁰The regret region is formally defined as:

$$R \equiv \{(y_{c2}, y_{p2}) \in [\gamma_c, \infty) \times [\gamma_p, \infty) : y_{c2} \leq \bar{y}_{c2}(y_{p2})\}.$$

²¹As such, it is defined over the same domain as Δ_2 , the set $[\gamma_c, \infty) \times [\gamma_p, \infty)$.

It is worth examining equation (7) in detail. First of all, the right-hand side is nonnegative. It represents the difference between expected utility under coresidence and under independence, i.e. the gain in expected utility associated with waiting for period 2 before choosing whether or not to move out. This is the option value. It will be strictly positive if the cdf $F(\cdot)$ places strictly positive mass on the regret region. The left-hand side represents the difference in period 1 utility from being independent relative to moving out. The child will move out when this gain exceeds the expected benefit from waiting. Note that the left-hand side is a difference between the within-period utility across residential states. It can be shown that this difference corresponds exactly to the function $\Delta_2(\cdot)$, only now the arguments of Δ_2 are the *first-period* incomes of child and parent.²² The results outlined in Lemmas 1 and 2 showed how $\Delta_2(\cdot)$ varied with second-period incomes. Those results carry over to period 1, establishing how the left-hand side of equation (7) varies with first-period incomes.

Define \bar{R} as the expected value of regret, the difference in expected utility between the best residential state (coresidence) and independence over the regret area. (\bar{R} is a notational shortcut to represent the right-hand side of (7).) Let \bar{y}_{c1} denote the first-period income threshold such that the child is exactly indifferent between staying at the parental income or moving out. This income level is such that (7) holds at equality:

$$\Delta_2(\bar{y}_{c1}, y_{p1}) = \bar{R}. \quad (8)$$

We now discuss the determination of \bar{y}_{c1} .

It is useful to begin by recalling how the child's second-period indifference threshold \bar{y}_{c2} was determined and comparing it to (8). In the second period, the child simply evaluates the differential in utilities across residential states and, if she has not moved out in period 1, chooses to live where utility is highest. If $\Delta_2(y_{c2}, y_{p2}) > 0$, she moves out, otherwise she stays, and \bar{y}_{c2} is such that she is just indifferent: $\Delta_2(\bar{y}_{c2}) = 0$. In

²²The equivalence between the left-hand side of (7) and $\Delta_2(\cdot)$ follows from noticing that the transfer function that governs transfers from parents to children in period 1, $t(y_{c1}, y_{p1})$, is identical to the function previously derived for period 2, $t(y_{c2}, y_{p2})$, once period 2 incomes are replaced with period 1 income values.

period 1, as illustrated in (8), she will require that utility while independent exceed coresidence utility by a strictly positive amount, \bar{R} . Therefore, while \bar{y}_{c2} was determined as the child's second-period income that set Δ_2 equal to zero, \bar{y}_{c1} is now the value of the child's first-period income that sets Δ_2 equal to \bar{R} . In view of the possibility of regret, in the first-period the child will demand that independence be strictly better than coresidence. Graphically, if we go back to Figure 1A, \bar{y}_{c2} was found by identifying the intercept of Δ_2 with the horizontal axis while \bar{y}_{c1} is now given by the intersection of Δ_2 with a horizontal line lying strictly above that axis. This discussion intuitively shows that the child's first-period moving out threshold will exceed \bar{y}_{c2} only if \bar{R} is positive. It will be shown below that, as long as $\alpha < 1$, the equation $\Delta_2(\cdot) = \bar{R}$ always has a root; further, since under $\alpha < 1$, $\Delta_2(\cdot)$ is strictly increasing for $y_{c2} \geq \bar{y}_{c2}$, this root is unique. However, for $\alpha > 1$, $\Delta_2(\cdot)$ will eventually have a decreasing range, converging to 0 as $y_{c2} \rightarrow \infty$. In this case, it could happen that the horizontal line \bar{R} does not intercept the function Δ_2 . This means that the child will never choose to leave home as expected regret is too high. If \bar{R} is low enough to intercept $\Delta_2(\cdot)$, then generally the equation $\Delta_2(\cdot) = \bar{R}$ has two roots. (One root would obtain if the horizontal line \bar{R} were tangent to the function Δ_2 .)

This discussion informally establishes the following result:

Proposition 3 (Expected regret and moving-out decision) *When nonempty, the period 1 moving-out threshold correspondence $\bar{y}_{c1}(y_{p1})$, on (y_{c1}, y_{p1}) space, lies strictly to the right of the corresponding period 2 schedule $\bar{y}_{c2}(y_{p2})$ if and only if $F(R) > 0$. When $\alpha < 1$, $\bar{y}_{c1}(y_{p1})$ exists and is single-valued.*

In what follows, we assume $\alpha < 1$. Below, we discuss alternative ways of ensuring that $\Delta_2(\cdot, y_{p2})$ is strictly monotonic for $y_{c2} \geq \bar{y}_{c2}$. Further, we also confine attention to the case when \bar{R} is strictly positive (for otherwise the moving-out decision in period 1 would be identical to that of period 2).

Our next step is to characterize how the child's residential choice depends on future income, hers and her parents'. For example, if the child suddenly received the good

news that her expected income in period 2 was going to be higher, would \bar{y}_1 increase or decrease? What if the good news were about her parents' income instead? We will consider two types of changes in the distribution of future income values; specifically, we will allow the distributions of future income to shift in the sense of first- and second-order stochastic dominance.

First-Order Stochastic Dominance We say that distribution $F^1(x)$ dominates $F^2(x)$ in the first-order stochastic sense if

$$F^1(x) \leq F^2(x), \forall x.$$

Shifts in the distribution of future incomes affect the residential choice as described in (8) to the extent that they modify the expected value of regret, \bar{R} . In turn, \bar{R} is the (negative of the) expected value of the values of Δ_2 over income pairs (y_{c2}, y_{p2}) in the regret area. For example, say that y_{p2} is in fact constant. Then, \bar{R} equals (minus) the expectation of the values of Δ_2 for y_{c2} in the interval $[\gamma_c, \bar{y}_{c2}]$. Figure 1A shows one configuration for Δ_2 . While in that Figure Δ_2 is strictly monotonic over the relevant interval, this need not be the case at all, as Lemma 1 illustrates. If Δ_2 were monotonically increasing over the regret area, it would be straightforward to show that a shift in the distribution of the child's future income in the first-order stochastic sense would reduce \bar{R} and, as a consequence, reduce \bar{y}_{c1} , as well. Since our results hinge crucially on the lack of monotonicity of $\Delta_2(\cdot)$ in the range $y_c \in [\gamma_c, \bar{y}_{c2}]$, we next go over the factors that determine the slope of Δ_2 in some detail.

Since Δ_2 corresponds to a difference in utility levels, changes in income affect this difference in two ways. First, income modifies consumption differently depending on the residential state. For example, for y_{c2} values such that no transfers would be provided to the child (i.e. above \tilde{y}_{c2}), higher y_{c2} implies that c_{i2} is changing by the same amount as income, whereas the increment in consumption at the parental home is only the fraction $1/n$ of the change in income. We label the impact of income changes on the child's consumption as the *sharing effect*. This, however, is not sufficient to ensure that Δ_2

varies positively with y_{c2} . The impact on Δ_2 depends also on the marginal utility that these changes in consumption entail. If, for example, $c_{i2} > c_{p2}^n$, the marginal utility of consumption at home is higher than under independence. In the range $y_{c2} > \bar{y}_{c2}$, this *marginal utility effect* counteracts the greater change in c_{i2} relative to c_{p2}^n . Consequently, although we know that c_{i2} will always exceed c_{p2}^n provided $y_{c2} > \bar{y}_{c2}$, we cannot be certain that Δ_2 is always positively sloped in this range. When $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$, by contrast, both effects go in the same direction, ensuring that Δ_2 is positively sloped. As discussed above, $\alpha < 1$ is a sufficient condition to obtain the strict monotonicity of Δ_2 with respect to y_{c2} , when $y_{c2} > \bar{y}_{c2}$. More generally, what is needed is that, for high consumption values – high enough to justify independence – the sharing effect outweigh the marginal utility effect. This is a plausible assumption since the marginal utility from consumption at home is likely to be close to that under independence when consumption is high in both residential states.

For $y_{c2} < \tilde{y}_{c2}$, parents give transfers to independent children. Given partial altruism and housing costs, we know the child experiences lower consumption while independent relative to coresidence. Further, an extra dollar of the child’s income will be shared with her family through a reduction in parental transfers. Under partial altruism, consumption while independent will increase by less than the consumption the child would attain if she were at the parental home. However, since the child is worse off when independent, the marginal utility effect indicates that one unit of extra consumption will raise the utility of an independent child the most. For $y_{c2} \in [\gamma_c, \tilde{y}_{c2}]$, we have sharing and marginal utility effects going in opposite directions.

There are reasonable assumptions that would allow us to solve the ambiguity from the effects of higher income over Δ_2 , in the positive transfer region. For example, we could assume that the marginal utility effect dominates whenever income pairs (y_{p2}, y_{c2}) would trigger transfers to independent children. Since transfers are given when the differential between independent consumption and that experienced at the parental home is greatest, this is a plausible assumption. This assumption does ensure that $\Delta_2(y_{c2})$ is monotonically increasing and, as a consequence, that \bar{R} decreases following

a first-order shift in the child's second-period income distribution. As a function of the child's income, $\Delta_2(\cdot)$ would qualitatively look like Figure 1A. Interestingly, this assumption coupled with altruism then causes monotonicity to fail when we consider $\Delta_2(\cdot)$ as a function of parental income, as depicted in Figure 1B. The reason is that, for altruistic parents, higher parental income will also affect the child's independence utility provided the child is poor enough to receive transfers. For income low enough to trigger transfers, an extra dollar of parental income will have exactly the same impact over the utility differential – independence minus coresidence – as an extra dollar of the child's income. In fact, when transfers are positive, parents effectively choose the child's consumption by selecting the amount of the transfer they are giving her. The optimal choice of consumption, for y_c and y_p values that trigger transfers, depends only on the sum $y_c + y_p$ and not on its individual parcels. This is an instance of Ricardian Equivalence type of neutrality results. This implies that, while for $y_{p2} < \tilde{y}_{p2}$, higher parental income will unambiguously reduce the utility differential from independence (as a function of y_{p2} , Δ_2 is strictly decreasing as illustrated in Figure 1B), for values of y_{p2} that exceed \tilde{y}_{p2} so that positive transfers occur, the slope of $\Delta_2(y_{p2})$ will equal the slope of $\Delta_2(y_{c2})$ and, according to the configuration displayed in Figure 1A, raise it.

This discussion informally establishes the result that, if $\Delta_2(\cdot)$ is monotonic in the child's income, such monotonicity will fail when we consider $\Delta_2(\cdot)$ as a function of parental income. We formalize this result as follows:

Lemma 4 (Altruism and the lack of monotonicity in differential utility) *For $\lambda \in [0.5, 1)$, if the function $\Delta_2(\cdot, y_{p2})$ is strictly monotonic with respect to the child's income, then $\Delta_2(y_{c2}, \cdot)$ cannot be strictly monotonic with respect to the parents' income. The converse is also true: if $\Delta_2(y_{c2}, \cdot)$ is strictly monotonic with respect to the parents' income, then $\Delta_2(\cdot, y_{p2})$ cannot be strictly monotonic with respect to the child's income.*

When $\lambda = 1$, parents are selfish and place no value on the child's utility. This is one instance where the monotonicity of $\Delta_2(\cdot)$ with respect to both the child and the parents' income can be obtained and we discuss this case below.

Given Lemma 4, unambiguous results concerning the impact of shifts in the distribution of future incomes in the first-order stochastic sense can only be obtained by considering the subset of distributions of (y_{c2}, y_{p2}) that place no mass on the subset of R where transfers are positive. This is summarized in the following propositions.

Let \mathcal{F} be the set of all pairs of independent distributions functions (F_c, F_p) with support over $([y_c, \infty), [y_p, \infty))$, such that no mass is placed on the positive-transfer subset of the regret region. Then:

Proposition 5 (FOSD in the child's income) *Let (F_p, F_c^1) and (F_p, F_c^2) be two elements of \mathcal{F} , and assume that F_c^1 first-order stochastically dominates F_c^2 . Let the period 1 moving-out threshold corresponding to F_c^j be denoted $\bar{y}_{c1}(F_c^j)$. Then, when $\alpha < 1$, $\bar{y}_{c1}(F_c^1) \leq \bar{y}_{c1}(F_c^2)$.*

Proposition 6 (FOSD in the parent's income) *Let (F_p^1, F_c) and (F_p^2, F_c) be two elements of \mathcal{F} , and assume that F_p^1 first-order stochastically dominates F_p^2 . Let the period 1 moving-out threshold corresponding to F_p^j be denoted $\bar{y}_{c1}(F_p^j)$. Then, when $\alpha < 1$, $\bar{y}_{c1}(F_p^1) \geq \bar{y}_{c1}(F_p^2)$.*

Next, we briefly sketch how our results would change in two scenarios, the opposing cases of full altruism ($\lambda = 0.5$), and of no altruism ($\lambda = 1$). Under full altruism, an extra dollar of income (either the parents' or the child's) would have the same impact on the child's consumption irrespective of her residential choice.²³ As such, there is no differential sharing effect. This ensures that $\Delta_2(y_{c2})$ is unambiguously positively sloped for $y_{c2} \leq \tilde{y}_{c2}$, and that $\Delta_2(y_{p2})$ is also positively sloped for $y_{p2} \geq \tilde{y}_{p2}$. While proposition 5 could be generalized to consider any joint distribution $F(\cdot)$ (and not only those who place no mass over the positive-transfer subset of the regret area), the same ambiguity as above would emerge when considering the effects of parental income on coresidence. Therefore, results in this case would be qualitatively similar to those in Propositions 5 and 6, above.

²³As mentioned earlier, the sharing rule prevailing at the parental home is equivalent to full altruism.

If parents were completely selfish, transfers would never be given out and the child would only be able to share the income of her family members under coresidence. In the case of selfish parents, the transfer region vanishes and \tilde{y}_{c2} coincides with γ_c . From the point of view of Figure 1A, the interval $[\gamma_c, \tilde{y}_{c2})$ ceases to exist and, from Lemma 1, it follows that $\Delta_2(y_{c2})$ is strictly increasing for $y_{c2} \in [\tilde{y}_{c2}, \bar{y}_{c2}]$ (sharing and marginal utility effects work in the same direction, here). Further, $\Delta_2(y_{p2})$ would also be monotonically decreasing everywhere (the threshold \tilde{y}_{p2} becomes infinity, now). An unintuitive conclusion follows from the comparison between the altruism and nonaltruism cases, the fact that altruism is the source of the potential ambiguity in the effects of shifts in the distribution of future incomes of parent and child. Absent altruism, the intuitive result that higher expected income of the child makes her more willing to leave (lower \bar{y}_{c1}) and that higher expected income of the parent has the opposite effect (higher \bar{y}_{c1}) would follow.

An additional comparison concerning the intensity of parental altruism is possible. Children of more altruistic parents will receive higher transfers when independent than children of less altruistic progenitors. Consequently, consumption while independent in the positive-transfer income region will always be negatively related to λ , the degree of parental selfishness, and the utility differential between independence and staying home will be less negative for children of more altruistic parents, in this region. (More generally, the function $\Delta_2(\cdot)$ for a child of more altruistic parents is everywhere above that of a child of a less caring family for income values such that the former would receive transfers; they overlap for other income values.) For children of more altruistic parents, therefore, regret will be less severe. It follows that the income threshold for independence for these children is lower than for those with more selfish progenitors. In this sense, and holding other things constant, “love” will push children out by making independence – and therefore future regret – less painful. Children of more selfish parents stay home so they can extract by presence what children of more loving progenitors get by voluntary parental transfers. As we have shown, however, while greater parental altruism unambiguously improves the prospects of independence, the presence of altru-

ism introduces ambiguous effects on how expected regret changes once future income prospects are modified.

Second-Order Stochastic Dominance We have seen how income insecurity, as measured by FOSD, affects the child's residential choice. One well-known implication of FOSD is higher expected income (but possibly also higher income variance). By looking now at second-order stochastic dominance shifts (SOSD) in the income distribution, we hold the expected value of income constant and see instead what happens when only the variance changes. We say that $F_c^1(y)$ dominates distribution $F_c^2(y)$ in the second-order stochastic sense if: i) $\int y F_c^1(y) dy = \int y F_c^2(y) dy$, and ii) $\int_{\gamma_c}^{y_c} [F_c^1(z) - F_c^2(z)] dz \leq 0$, with the inequality holding for all y_c in the domain of the child's income.²⁴ In other words, income becomes less volatile. Once again, the lack of monotonicity in $\Delta_2(y_c)$ and $\Delta_2(y_c)$ has implications for the concavity of these functions over the positive-transfer region. In order to get unambiguous results, we are again forced to restrict our results to income distributions that place no mass on that part of the income domain. For the child, we get the following result:

Proposition 7 (SOSD in the child's income) *Let (F_p, F_c^1) and (F_p, F_c^2) be two elements of \mathcal{F} such that F_c^1 dominates F_c^2 in the second-order stochastic sense. Then, when $\alpha < 1$, $\bar{y}_{c1}(F_c^1) \leq \bar{y}_{c1}(F_c^2)$.*

Under some conditions, a similar result can be obtained for SOSD shifts of parental income. However, since changes in the distribution of income affect the period 1 moving out threshold only to the extent that they affect period 2 income values within the regret area, for the parents' income we need to ensure that the distribution of his income shifts so as to lower the variance of income values in that region specifically, as opposed to the requirement that it becomes less volatile over its global range.²⁵ That is, if F_p^1

²⁴The results associated with first- and second-order stochastic dominance require the additional assumption that income is bounded. That is, $y_{p2} \leq y_p^* < \infty$ and $y_{c2} \leq y_c^* < \infty$, so that $F_p(\gamma_p) = F_c(\gamma_c) = 0$ and $F_p(y_p^*) = F_c(y_c^*) = 1$. We omit making this assumption explicit for simplicity.

²⁵For the child, the SOSD requirement that income becomes less volatile over its entire domain necessarily implies that it also becomes less volatile over the no-transfer part of the regret region. The

dominates F_p^2 in the second-order stochastic sense, we need additionally to impose that, for all values of the child's income y_{c2} ,

$$\int_{\gamma_p}^{\hat{y}_{p2}(y_{c2})} [F_p^1(y_{p2}) - F_p^2(y_{p2})] dy_{p2} = 0, \quad (9)$$

which, together with the conditions that ensure the second-order stochastic dominance of F_p^1 over F_p^2 , implies:

$$\int_{\hat{y}_{p2}(y_{c2})}^{y_{p2}} [F_p^1 - F_p^2] dy_{p2} \leq 0,$$

for all $y_{p2} \geq \hat{y}_{p2}(y_{c2})$.

We may then state:

Proposition 8 (SOSD in the parent's income) *Let (F_p^1, F_c) and (F_p^2, F_c) be two elements of \mathcal{F} satisfying equation (9), such that F_p^1 dominates F_p^2 in the second-order stochastic sense. Then, when $\alpha < 1$, $\bar{y}_{c1}(F_p^1) \geq \bar{y}_{c1}(F_p^2)$.*

The same generalizations concerning the degree of altruism stated for FOSD carry over to SOSD. Specifically, under full altruism, the results would be qualitatively similar to propositions 7 and 8 (and income distributions would have to be restricted so as to place no mass on the positive transfer region). If the parents were selfish, then shifts in the SOSD sense of the child's (parents') income distribution would unambiguously lower (raise) the first-period income moving-out threshold, the latter statement conditional on condition (9) holding.

3.1 Other Results and Generalizations

While the main focus of the paper is the impact of income insecurity on the child's residential decision, our model allows us to make additional predictions. We list below

reason is that, for the child, the no-transfer region coincides with the lowest possible realizations of the child's income. As such, the requirement that $\int_{\gamma_c}^{y_c} [F^1(y_c) - F^2(y_c)] dy_c \leq 0$ must also hold for any income value y_c in the regret region. For the parent, the regret region occurs for high y_p values. Therefore, the requirement $\int_{\gamma_p}^{y_p} [F^1(y_p) - F^2(y_p)] dy_p \leq 0$ does *not* imply that the latter condition holds within the regret region. That is, SOSD does not imply, for y within the regret region, that $\int_{\hat{y}_p}^y [F^1(y_p) - F^2(y_p)] dy_p \leq 0$ holds.

comparative static results concerning the impact of first-period parental income, y_{p1} , and family size, n , on the child's first-period moving-out threshold.²⁶

Lemma 9 (Parental income and coresidence) *When $\alpha < 1$, higher period 1 parental income y_{p1} raises the child's moving-out threshold $\bar{y}_{c1}(y_{p1})$.*

Lemma 10 (Family size and coresidence) *When $\alpha < 1$ and $\lambda = 0.5$ (very altruistic parents), for $n^+ > n$, the first period moving-out threshold is lower for a member of a more numerous family than the corresponding threshold for a child belonging to a smaller family:*

$$\bar{y}_1^{n^+} < \bar{y}_1^n.$$

Reversible Independence (or Finite Moving Costs) We assumed independence to be an irreversible state. An alternative but fully equivalent interpretation is for the cost of moving back home to be infinite. How would our results change if the child could pay some finite cost ϕ in order to go back to the parental home?

Second-period decisions would not change, naturally. As for the first period, the possibility of going back home would imply that the child would not wish to remain independent for second-period income realizations associated with a very negative value of Δ_2 . Under the configuration of Δ_2 displayed in Figure 1A, these would be associated with the lowest income realizations for the child. Given that the child could exercise her option of returning home (and would do so for ϕ sufficiently small and y_{c2} sufficiently low), expected regret \bar{R} would now be strictly lower than before. FOSD and SOSD results would follow just as before, also with the same caveats – caveats that have to

²⁶Concerning the effects of parental income, our results relate to Ermisch (2003) as follows. He studies coresidence choices under a static model, and considers both selfish and altruistic parents. Under the former case, parents charge the child for living at home. Coresidence is beneficial for the child since she gets to share housing, a public good. The child's utility when independent limits the amount of "rent" that the parents can charge her. When parental income increases, parents consume more of both direct consumption goods and housing. Since they provide a bigger house to the child, they also raise the rent they charge her. This reduces the probability of the child remaining at home. In our setup, Lemma 9 would go through even in the case of selfish parents: given the technology for sharing resources at home, the child still benefits from higher consumption when coresiding if her parents' income goes up. The predictions of our model and those in Ermisch coincide when altruistic parents are considered.

do with the lack of strict monotonicity in both $\Delta_2(\cdot, y_{p2})$ and $\Delta_2(y_{c2}, \cdot)$. Our algebra and analysis could therefore fully accommodate a finite ϕ . We choose to stick to infinite moving costs only for the sake of analytical simplicity.

3.2 The Special Case of Two-Tier Labor Markets

The analysis so far shows the difficulties in obtaining general results. In particular, most results were derived under the condition that the positive-transfer subset of the regret area receives no positive mass from the joint income distribution of parents and child. This restriction boils down to assuming that children will get high enough wages so that their parents would want to collect transfers *from* them.

Another special case of interest concerning the distribution of income – one that does not require the previous assumption – is the prevalence of a two-tier labor market in the following sense. Suppose children (or parents) may find two types of jobs: high-paying and rather permanent occupations versus low-paying jobs with easy dismissal. Suppose further that the low-paying job entails regret. Then, children effectively face insecure jobs under which they would prefer to stay home, or safe jobs which enable independence.

We may formalize this situation as follows. Let the child's income y_c take on two values, $y_c \in \{y_L, y_H\}$, with $y_H > y_L$, and where y_L occurs with probability π_c . A decline in π_c represents a FOSD shift in the child's income distribution. For simplicity, and without any loss of generality, we assume that parental income is constant at y_p . We further assume that y_L is associated with regret: an independent child would prefer to be at home if her pay equalled y_L .²⁷ This implies that:

$$c_i(y_L, y_p) - c_p^n(y_L, y_p) < 0.$$

The high income realization, on the other hand, entails no regret:

$$c_i(y_H, y_p) - c_p^n(y_H, y_p) > 0.$$

²⁷In this context, y_L could be low enough to fall within the positive-transfer subset of the regret region.

In this case, expected regret is simply

$$\bar{R} = \pi_c (u(c_p^n(y_L, y_p)) - u(c_i(y_L, y_p))) > 0, \quad (10)$$

and

$$\bar{R} = \bar{R} \left(\frac{\pi_c}{\bar{\pi}_c} \right).$$

Thus, expected regret is always positive for $\pi_c > 0$ but converges monotonically to 0 as $\pi_c \rightarrow 0$. We further assume that, when π_c is very high (e.g. unity), expected regret is too high and the child does not move out. For $\pi_c = 1$, for example, this implies that

$$u(c_p^n(y_L, y_p)) - u(c_i(y_L, y_p)) > u(c_i(y_H, y_p)) - u(c_p^n(y_H, y_p)).$$

Thus, for values of π_c close to unity, the utility gain upon receiving y_H is not high enough to induce the child to move out.

As a function of experience and repeated job market participation, the child will eventually see a reduction in π_c and start earning y_H with high probability. From the properties of \bar{R} , it follows that there is a positive but low enough π_c , $\bar{\pi}_c$, such that

$$\Delta_2(y_H, y_p) = \bar{\pi}_c (u(c_p^n(y_L, y_p)) - u(c_i(y_L, y_p))) = \bar{R}(\bar{\pi}_c).$$

Further, for any $\pi_c < \bar{\pi}_c$, the utility differential between independence and coresidence experienced under y_H exceeds expected regret to be suffered in the following period.

Therefore, when faced with y_L , the child does not wish to move out for coresidence is currently better than independence and there is the added possibility of regret in the future. But when faced with the high income value, if π_c is lower than $\bar{\pi}_c$, today's utility differential after moving out exceeds expected regret, and the child leaves the parental home.

The following result follows immediately from the previous analysis:

Proposition 11 *For $\alpha < 1$, the child's first-period moving out threshold \bar{y}_{c1} is monotonically increasing in the probability of the low income realization π_c . The child does not move out if her income is y_L but, for $\pi_c \leq \bar{\pi}_c$, the child will move out in the first period if she draws the high income y_H .*

Therefore, for FOSD shifts summarized by changes in π_c , unambiguous results follow. The reason is that there is only one value of $\Delta_2(\cdot)$ to compute in the regret area, and this value depends only on the constant numbers y_L and y_H . Therefore, whether or not $\Delta_2(\cdot)$ is monotonic or not is of no importance here. Higher π_c only raises the weight attached to the utility differential – independence minus coresidence – in the computation of \bar{R} , but not the value of the utility differential itself.

We can easily generalize the results for the parent as well. Suppose that now it is the child's income that is constant at y_c (again this carries no loss of generality). Parental income takes on two values, $\{y_{pL}, y_{pH}\}$, and π_p denotes the probability of the low income draw. We assume:

$$c_i(y_c, y_{pL}) - c_p^n(y_c, y_{pL}) > 0, c_i(y_c, y_{pH}) - c_p^n(y_c, y_{pH}) < 0,$$

so that the child experiences regret if she moves out and has wealthy parents, but she would prefer to be independent if her parents are poor. Expected regret is now:

$$\bar{R} = (1 - \pi_p) (u(c_p^n(y_c, y_{pH})) - u(c_i(y_c, y_{pH}))) = \bar{R}(\bar{\pi}_p),$$

and it varies inversely with π_p , converging to 0 as $\pi_p \rightarrow 1$.

As above, we assume that, for low values of π_p , close to zero, the child would prefer to stay home. When $\pi_p = 0$, this would imply:

$$u(c_i(y_c, y_{pL})) - u(c_p^n(y_c, y_{pL})) < u(c_p^n(y_c, y_{pH})) - u(c_i(y_c, y_{pH})).$$

Thus, a child whose parents were to receive the high income in period 2 would not find it worthwhile to leave even though her current utility from coresiding with low income parents is less than that of contemporaneous independence.

Let $\bar{\pi}_p$ be such that

$$\Delta_2(y_c, y_{pL}) = (1 - \bar{\pi}_p) (u(c_p^n(y_c, y_{pH})) - u(c_i(y_c, y_{pH}))) = \bar{R}(\bar{\pi}_p).$$

For any $\pi_p \geq \bar{\pi}_p$, the child of a poor father will move out. The following result follows immediately.

Proposition 12 *For $\alpha < 1$, the child's first-period moving out threshold \bar{y}_{c1} is monotonically decreasing in the probability of the low parental income realization, π_p . The child will not move out when her parents' income is y_{pH} but, for $\pi_p \geq \bar{\pi}_p$, the child will move out in the first period if the parents draw the low income value y_{pL} .*

The comparison of income distributions according to SOSD requires that expected income be kept constant. Therefore, a reduction in π_c (or π_p) is not suited for that purpose as it raises expected income. An alternative would be to consider a mean-preserving spread, whereby both values of y_L and y_H would converge to the distribution's expected value.²⁸ Unfortunately, the ubiquitous lack of monotonicity of the $\Delta_2(\cdot)$ function from before will now also come into play as we cannot unambiguously sign how expected regret changes as y_L increases (except if y_L is confined to the no-transfer part of the regret region, which falls into the general case outlined above and defeats the purpose of the current section).²⁹ Therefore, no straightforward generalization is available for the SOSD case here.

What are the empirical predictions of our model under the two-tier labor-market interpretation? In this case, children will only leave home once they find a stable, high-paying job; under these circumstances, they will receive no transfers. Countries such as Italy and Spain have been informally characterized as having such a two-tier labor market.³⁰ There, emancipation age is very high and the incidence of transfers between parents and their adult children appears to be very low, as follows. Guiso and Jappelli (2002) for the 1991 wave of the Survey of Household Income and Wealth (SHIW) indicate

²⁸Using the child as an example, the original distribution with income values $\{y_L, y_H\}$ would be a mean preserving spread of another distribution where the low income value were higher than y_L and the high income value lower than y_H , and where the low income realization took place with probability π_c and the expected value of income remained identical across both distributions.

²⁹One alternative which would deliver unambiguous results regarding SOSD is as follows. Again using the child as an example, consider a new income distribution where y_L would remain constant but be drawn with smaller and smaller probability, and where y_H converged to the mean. The probability of the low income and the value of the high income would be chosen so as to keep expected income constant. This new distribution would dominate the original one in the SOSD sense. Since y_L is kept constant, expected regret again depends only on the probability that the low income is drawn; high income does not affect expected regret as it leads the child to independence. But this special case is uninteresting since it exactly mimics the FOSD case above.

³⁰See e.g. Bentolila and Dolado (1994) for Spain.

that only 5.6% of households in their sample reported receiving a transfer from their parents or other relatives whereas the corresponding figure for Spain, from Bentolila and Ichino (2008), is 5%.³¹ In this case, going from the insecure low-paying job to the stable high-paying occupation amounts to a FOSD shift in the child's income; further, it also entails going from an income distribution with regret to one without it. In this case, FOSD shifts in income unambiguously favor independence.

What predictions would the analysis have for a labor market where there are no secure jobs but unemployment duration is very low, so that switching to a more desirable occupation in a short time is a feasible prospect? This could perhaps correspond to the US. In this case, we would expect children to be less reluctant to leave (there is no safe job to wait for, after all) but, since independent children would become unemployed more frequently, to also observe a greater incidence of transfers. In the US, according to Villanueva (2005) who uses the 1988 wave of the Panel Study of Income Dynamics (PSID), the percentage of parent-child pairs for which transfers were given was 31% (Table 5).³²

Empirical Support In related work (Becker, Bentolila, Fernandes, and Ichino (2008)), we estimate the effect of perceived parental income insecurity on the child's probability of staying home. We explore the panel dimension of the SHIW and follow families in two consecutive sample dates, 1995 and 1998. In 1995, respondents were asked to state the probability they assigned to keeping their current job, if they were employed, or of finding a new job, if unemployed. We use the complement of this probability, which we label p . Our empirical work provides estimates of the impact of changes in p on the child's probability of coresidence. The dependent variable is a 0-1 indicator of whether a

³¹Bentolila and Ichino (2008) use the *Encuesta Continua de Presupuestos Familiares* for Spain. They also report a slightly higher transfer incidence for Italy, of 9%, using the same panel as Guiso and Japelli.

³²Other studies report a significantly lower incidence of transfers for the US. For example, in Bentolila and Ichino (2008), transfer incidence in regular PSID samples is of only 3%. They suggest that the substantially higher transfer incidence reported in other studies that use the PSID transfer supplement may be due to the properties of that source. It is therefore likely that transfer incidence across these different studies is not directly comparable.

child was still home in 1998, given that she was home in 1995. In the regressions, we use other controls as well: father’s age and schooling, gender of the child, number of siblings, measures of housing prices, and objective measures of job insecurity (such as the change in the fraction of unemployed workers between the current and future calendar year for a given age-bracket, gender and province cell, as well as the change in the fraction of temporary jobs for the same age-gender-location category).

We find the inclusion of p to be a valid control for FOSD, as follows. To the extent that father’s age and schooling control for the father’s income level when employed, and since unemployment benefits are proportional to previous wages in Italy, the degree of perceived job insecurity measures (the complement of) the probability that the parent will get his full wages, as opposed to the corresponding unemployment benefits. For this two-point support distribution of parental income (employment wages versus unemployment benefits), a reduction in perceived job insecurity (lower p) exactly captures the notion of first-order stochastic dominance used in the model. (It corresponds exactly to a reduction in π_p in the two-tier labor market discussion above).

We find strong and statistically significant effects of perceived parental income insecurity on the probability of independence. Specifically, if the parent’s perceived probability of becoming (or remaining) unemployed went from 0 (full job security) to 1 (full job insecurity), the child’s probability of becoming independent would increase by 1.7 percentage points. Taking into account that the average probability of independence in our sample is only 4%, these are considerable effects. Further, the two-tier labor market interpretation is consistent with our results in the following sense. For the parental cohort, it is the change in the fraction of unemployed workers that comes out as significant whereas for the children, it is the change in the fraction of temporary jobs that is so. It thus appears that job instability is relevant for the young’s residential choice whereas, regarding their parents, the young are concerned with whether or not parents will have a (stable) job.

The micro data further allows us to test the model’s predictions concerning SOSD

shifts in the distribution of parental income.³³ The coefficient of variation of the distribution of parental income has a strong and statistically significant effect over the probability of coresidence, as our model predicts. If the coefficient of variation went from 0 to 50% of the mean, the child's probability of coresidence would increase by 0.6 percentage points.

In our companion paper, we additionally test the impact of job insecurity on coresidence at the macro level for a sample of European Union countries. Data on job insecurity comes from the European Commission's Eurobarometer. Once again, we find strong and statistically significant effects of job insecurity on coresidence. If the percentage of youth feeling insecure went from 0 to 100, the coresidence rate would increase by about 17 percentage points; an identical change in the share of old workers feeling insecure would generate a decrease in the coresidence rate of about 11 percentage points. These are sizable effects when compared to the average coresidence rate in our sample of 48%. We see the joint micro and macroeconomic evidence as offering support to the relevance of income security as an important determinant of residential decisions.

4 Conclusion

We have analyzed how income insecurity affects the residential choice of a young adult under a model of partial altruism and costly independence. Our results show that first- and second-order stochastic dominance shifts in the distribution of the child's (or the parents') future income do not necessarily produce the intuitive result of reducing (raising) the child's income threshold for independence. Altruism is the very source of this ambiguity. For selfish parents or if financial transfers between parents and their adult children occur with very low probability (the latter effectively shutting down the range of income values where altruism is operative), these results do emerge. Similar conclusions follow for second-order stochastic dominance shifts in the income distribution: lower variance in the child's (parents') future income raises (reduces) the child's threshold

³³We get our measures of income uncertainty from Guiso, Jappelli and Pistaferri (2002).

for independence provided transfers are not operative. Empirical estimates using panel data for Italy and macroeconomic data for European Union countries broadly confirm our predictions.

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A Proofs

Proof of Lemma 1

We first note that:

$$\Delta_2 > 0 \Leftrightarrow c_{i2} > c_{p2}^n.$$

In words, the child prefers to move out if and only if her consumption while independent exceeds the consumption she would enjoy if she stayed. When parental income is such that $\tilde{y}_{c2} > \gamma_c$ (so that there is a range of incomes for the child for which she would receive strictly positive parental transfers), the difference in the child's consumption across residential states is:

$$c_{i2} - c_{p2}^n = \frac{y_{p2} + y_{c2} - \gamma_p - \gamma_c}{(\Gamma(n-1) + 1)} - \frac{y_{p2} + y_{c2} - \gamma_p}{n},$$

which is negative, since $\Gamma(n-1) + 1 > n$. This shows that $\Delta_2(y_{c2}) < 0$ for $y_{c2} \in [\gamma_c, \tilde{y}_{c2}]$. For $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2}]$, the child would not receive any transfers if she moved out. In this case,

$$c_{i2} - c_{p2}^n = y_{c2} - \gamma_c - \frac{y_{p2} + y_{c2} - \gamma_p}{n}.$$

It is straightforward to show that this difference is negative for income values y_{c2} such that $y_{c2} < \bar{y}_{c2}$, and positive for $y_{c2} > \bar{y}_{c2}$. This proves that $\Delta_2(y_{c2}) < 0$ for $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$ and $\Delta_2(y_{c2}) > 0$ for $y_{c2} > \bar{y}_{c2}$.

The derivative of Δ_2 with respect to y_{c2} is:

$$\frac{\partial u(c_{i2})}{\partial c_{i2}} \frac{\partial c_{i2}}{\partial y_{c2}} - \frac{\partial u(c_{p2}^n)}{\partial c_{p2}^n} \frac{\partial c_{p2}^n}{\partial y_{c2}}.$$

For $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$, $\partial c_{i2}/\partial y_{c2} = 1$, and $\partial c_{p2}^n/\partial y_{c2} = 1/n$. Also, since $\Delta_2 < 0$ in this range, $\partial u(c_{i2})/\partial c_{i2}$ exceeds $\partial u(c_{p2}^n)/\partial c_{p2}^n$. This implies that $\partial \Delta_2/\partial y_{c2} > 0$ in this interval.

The expression for $\partial \Delta_2/\partial y_{c2}$, for $y_{c2} \in (\gamma_c, \tilde{y}_{c2})$ is given by:

$$\frac{\partial \Delta_2}{\partial y_{c2}} = (c_{i2})^{-\alpha} \frac{1}{(\Gamma(n-1) + 1)} - (c_{p2}^n)^{-\alpha} \frac{1}{n},$$

and

$$\begin{aligned} \frac{\partial \Delta_2}{\partial y_{c2}} > 0 &\Leftrightarrow \left(\frac{c_{p2}^n}{c_{i2}} \right)^\alpha > \frac{\Gamma(n-1) + 1}{n} \\ &\Leftrightarrow \left(\underbrace{\frac{y_{p2} + y_{c2} - \gamma_p}{y_{p2} + y_{c2} - \gamma_p - \gamma_c}}_{>1} \right)^\alpha > \left(\underbrace{\frac{(\Gamma(n-1) + 1)}{n}}_{>1} \right)^{1-\alpha}. \end{aligned}$$

Since $\Gamma > 1$, the term in braces on the right-hand side exceeds unity. When $\alpha > 1$, the right-hand side will be smaller than 1 and, since the left-hand side of the inequality is greater than 1, the inequality will be satisfied for all values of y_{p2} and y_{c2} . This proves that $\Delta(y_{c2})$ is strictly increasing for $y_{c2} \in (\gamma_c, \tilde{y}_{c2})$ when $\alpha > 1$.

The expression for $\partial \Delta_2 / \partial y_{c2}$, for $y_{c2} > \bar{y}_{c2}$ is given by:

$$\frac{\partial \Delta_2}{\partial y_{c2}} = (c_{i2})^{-\alpha} - (c_{p2}^n)^{-\alpha} \frac{1}{n},$$

and

$$\frac{\partial \Delta_2}{\partial y_{c2}} > 0 \Leftrightarrow \left(\frac{c_{p2}^n}{c_{i2}} \right)^\alpha > \frac{1}{n} \Leftrightarrow \left(\underbrace{\frac{y_{p2} + y_{c2} - \gamma_p}{y_{c2} - \gamma_c}}_{>1} \right)^\alpha > \left(\underbrace{\frac{1}{n}}_{<1} \right)^{1-\alpha}.$$

Since $y_{p2} \geq \gamma_p$, the fraction on the left-hand side of this inequality exceeds unity and, since families have at least two persons, the fraction on the right-hand side is smaller than 1. For $\alpha < 1$, therefore, the inequality above is satisfied for all values of y_{c2} and y_{p2} . This proves that $\Delta_2(y_{c2})$ is strictly increasing for $y_{c2} > \bar{y}_{c2}$ when $\alpha < 1$. ■

Proof of Lemma 2

The proof is identical to the previous one. ■

The proof of Proposition 3 builds on auxiliary Lemma A.0.

Lemma A.0 As $y_{c2} \rightarrow \infty$, we have that:

$$\lim_{y_{c2} \rightarrow \infty} \Delta_2(y_{c2}) = \begin{cases} \infty & , \text{ if } \alpha < 1 \\ 0 & , \text{ if } \alpha > 1 \end{cases}.$$

Proof of Lemma A.0

As $y_{c2} \rightarrow \infty$, eventually it enters the independence range where $\Delta_2 > 0$. In this case:

$$\begin{aligned} \lim_{y_{c2} \rightarrow \infty} \Delta_2(y_{c2}) &= \lim_{y_{c2} \rightarrow \infty} \{u(c_{i2}) - u(c_p^n)\} \\ &= \lim_{y_{c2} \rightarrow \infty} \left\{ \frac{(y_{c2} - \gamma_c)^{1-\alpha}}{1-\alpha} - \frac{1}{1-\alpha} \left(\frac{y_{p2} + y_{c2} - \gamma_p}{n} \right)^{1-\alpha} \right\}. \end{aligned}$$

When $\alpha > 1$, the previous expression converges to 0. When $\alpha < 1$, we have:

$$\begin{aligned} \lim_{y_{c2} \rightarrow \infty} \Delta_2(y_{c2}) &= \lim_{y_{c2} \rightarrow \infty} \left\{ \frac{(y_{c2} - \gamma_c)^{1-\alpha}}{1-\alpha} - \frac{1}{1-\alpha} \left(\frac{y_{p2} + y_{c2} - \gamma_p}{n} \right)^{1-\alpha} \right\} \\ &= \lim_{y_{c2} \rightarrow \infty} \frac{1}{1-\alpha} \left\{ \frac{1 - \left(\frac{1}{n}\right)^{1-\alpha} \left(\frac{y_{p2} + y_{c2} - \gamma_p}{y_{c2} - \gamma_c}\right)^{1-\alpha}}{\frac{1}{(y_{c2} - \gamma_c)^{1-\alpha}}} \right\} = \infty. \blacksquare \end{aligned}$$

Proof of Proposition 3

If $F(R) > 0$, the right-hand side of (8) is strictly positive. The moving-out income threshold \bar{y}_{c1} , such that $\Delta_1(\bar{y}_{c1}) = 0$, solves:

$$\begin{aligned} u(c_{i1}(\bar{y}_{c1})) - u(c_{p1}^n(\bar{y}_{c1})) &= \int_R (u(c_{p2}^n) - u(c_{i2})) dF(y_{c2}, y_{p2}) > 0 \\ \Leftrightarrow \Delta_2(\bar{y}_{c1}) &= \int_R (u(c_{p2}^n) - u(c_{i2})) dF(y_{c2}, y_{p2}) > 0. \end{aligned}$$

Applying Lemma 1 to Δ_2 , we know that it is strictly negative for $y_{c1} < \bar{y}_{c2}$, and strictly positive for $y_{c1} > \bar{y}_{c2}$. Further, from the properties of the utility function $u(\cdot)$, Δ_2 is continuous. Since $\Delta_2(\bar{y}_{c2}) = 0$, it follows that, for identical values of parental income across periods, $y_{p1} = y_{p2}$, the value of $\bar{y}_{c1}(y_{p1})$ that solves the previous equation – if it exists – must strictly exceed $\bar{y}_{c2}(y_{p1})$. If $\Delta_2(y_{c2})$ has a decreasing range for $y_{c1} > \bar{y}_{c2}$, there could be either multiple solutions to the previous equation or none. If, however, $\alpha < 1$, $\Delta_2(y_{c1})$ has a strictly positive slope for $y_{c1} > \bar{y}_{c2}$, as shown in Lemma 1. Further, as auxiliary Lemma A.0 shows, it converges to ∞ as $y_{c2} \rightarrow \infty$. In this case, the value of \bar{y}_{c1} that solves the previous equation exists and is unique. \blacksquare

Proof of Lemma 4

First, we show that the derivative of $\Delta_2(\cdot)$ with respect to y_{c2} in the range $y_{c2} \in [\gamma_c, \tilde{y}_{c2})$ is identical to the derivative of $\Delta_2(\cdot)$ with respect to y_{p2} in the range $y_{p2} > \tilde{y}_{p2}$. Notice that, in both income intervals, an independent child is receiving transfers. We have that:

$$\frac{\partial \Delta_2}{\partial y_{c2}} \Big|_{y_{c2} \in [\gamma_c, \tilde{y}_{c2})} = \frac{\partial}{\partial y_{c2}} \left[\frac{1}{1-\alpha} \left(\frac{y_{p2} + y_{c2} - \gamma_p - \gamma_c}{\Gamma(n-1) + 1} \right)^{1-\alpha} - \frac{(y_{p2} + y_{c2} - \gamma_p)^{1-\alpha}}{1-\alpha} \right],$$

whereas

$$\frac{\partial \Delta_2}{\partial y_{p2}} \Big|_{y_{p2} > \tilde{y}_{p2}} = \frac{\partial}{\partial y_{p2}} \left[\frac{1}{1-\alpha} \left(\frac{y_{p2} + y_{c2} - \gamma_p - \gamma_c}{\Gamma(n-1) + 1} \right)^{1-\alpha} - \frac{(y_{p2} + y_{c2} - \gamma_p)^{1-\alpha}}{1-\alpha} \right],$$

the same as above. From Lemma 1, we have that $\Delta_2(\cdot, y_{p2})$ is strictly increasing for $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$, whereas Lemma 2 shows that $\Delta_2(y_{c2}, \cdot)$ is strictly decreasing for $y_{p2} \in (\hat{y}_{p2}, \tilde{y}_{p2})$. Say that $\partial \Delta_2 / \partial y_{c2}$ in the range $y_{c2} \in [\gamma_c, \tilde{y}_{c2})$ is positive. Then, $\Delta_2(\cdot, y_{p2})$ will be monotonically increasing for $y_{c2} \leq \bar{y}_{c2}$. However, this implies that $\Delta_2(y_{c2}, \cdot)$ will be decreasing for $y_{p2} \in (\hat{y}_{p2}, \tilde{y}_{p2})$ and increasing for values of y_{p2} that exceed \tilde{y}_{p2} . Conversely, if $\partial \Delta_2 / \partial y_{c2}$ were to be negative in the range $y_{c2} \in [\gamma_c, \tilde{y}_{c2})$, then monotonicity of $\Delta_2(\cdot, y_{p2})$ would fail, whereas $\Delta_2(y_{c2}, \cdot)$ would be strictly decreasing for $y_{p2} \geq \hat{y}_{p2}$. ■

Proof of Proposition 5

$\bar{y}_{c1}(F_c^1)$ is the solution to the first line of the following equation:

$$\begin{aligned} \Delta_2(\bar{y}_{c1}(F_c^1), y_{p1}) &= \int_{\gamma_p} \left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} (u(c_{p2}^n) - u(c_{i2})) dF_c^1(y_{c2}) \right] dF_p(y_{p2}) \\ &= - \int_{\gamma_p} \left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \Delta_2(y_{c2}, y_{p2}) dF_c^1(y_{c2}) \right] dF_p(y_{p2}) \\ &= - \int_{\gamma_p} \left(\begin{array}{c} \Delta_2(y_{c2}, y_{p2}) F_c^1 \Big|_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \\ - \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} F_c^1(y_{c2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{c2}) dy_{c2} \end{array} \right) dF_p(y_{p2}) \\ &= \int_{\gamma_p} \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} F_c^1(y_{c2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{c2}) dy_{c2} dF_p(y_{p2}) \\ &\leq \int_{\gamma_p} \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} F_c^2(y_{c2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{c2}) dy_{c2} dF_p(y_{p2}) \\ &= \Delta_2(\bar{y}_{c1}(F_c^2), y_{p1}), \end{aligned}$$

where we are using $F_c^1(\tilde{y}_{c2}(y_p)) = F_c^2(\tilde{y}_{c2}(y_p)) = 0$, $\Delta_2(\bar{y}_{c2}(y_{p2}), y_{p2}) = 0$, $\Delta_2(\cdot, y_{p2})$ is an increasing function of the child's income in the range $(\tilde{y}_{c2}(y_{p2}), \bar{y}_{c2}(y_{p2}))$, as shown in Lemma 1; the inequality follows finally from the assumed first-order stochastic dominance of F_1^c over F_c^2 .

When $\alpha < 1$, it follows that:

$$\Delta_2(\bar{y}_{c1}(F_c^1), y_{p1}) \leq \Delta_2(\bar{y}_{c1}(F_c^2), y_{p1}) \Leftrightarrow \bar{y}_{c1}(F_c^1) \leq \bar{y}_{c1}(F_c^2). \blacksquare$$

Proof of Proposition 6

$\bar{y}_{c1}(F_p^1)$ is the solution to the first line of the following equation:

$$\begin{aligned} \Delta_2(\bar{y}_{c1}(F_p^1), y_{p1}) &= \int_{\gamma_c} \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} [-\Delta_2(y_{c2}, y_{p2})] dF_{p2}^1 dF_{c2} \\ &= - \int_{\gamma_c} \left[\begin{aligned} &\Delta_2(y_{c2}, y_{p2}) F_p^1(y_{p2}) \Big|_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \\ &- \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^1(y_{p2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) dy_{p2} \end{aligned} \right] dF_{c2} \\ &= - \int_{\gamma_c} \left[\begin{aligned} &\Delta_2(y_{c2}, \tilde{y}_{p2}(y_{c2})) F_p^1(\tilde{y}_{p2}(y_{c2})) \\ &- \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^1(y_{p2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) dy_{p2} \end{aligned} \right] dF_{c2} \\ &= - \int_{\gamma_c} \left[\begin{aligned} &\Delta_2(y_{c2}, \tilde{y}_{p2}(y_{c2})) - \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^1(y_{p2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) dy_{p2} \end{aligned} \right] dF_{c2} \\ &\geq - \int_{\gamma_c} \left[\begin{aligned} &\Delta_2(y_{c2}, \tilde{y}_{p2}(y_{c2})) - \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} F_p^2(y_{p2}) (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) dy_{p2} \end{aligned} \right] dF_{c2} \\ &= \Delta_2(\bar{y}_{c1}(F_p^2), y_{p1}) \end{aligned}$$

where we are using $F_p^1(\tilde{y}_{p2}(y_{c2})) = F_p^2(\tilde{y}_{p2}(y_{c2})) = 1$, $\Delta_2(y_{c2}, \hat{y}_{p2}(y_{c2})) F_p^1(\hat{y}_{p2}) = 0$ (either because $\hat{y}_{p2} = \bar{y}_{p2}$ and $\Delta_2(y_{c2}, \bar{y}_{p2}(y_{c2})) = 0$ or, when $\hat{y}_{p2} = \gamma_p$, $F_p^1(\gamma_p) = 0$). The inequality follows from the first-order stochastic dominance of F_p^1 over F_p^2 and the fact that Δ_2 is a decreasing function of y_{c2} in the range $[\hat{y}_{p2}(y_{c2}), \tilde{y}_{p2}(y_{c2})]$, as shown in Lemma 2. Given $\alpha < 1$ and the corresponding strict monotonicity of $\Delta_2(\cdot, y_{p1})$ for $y_{c1} \geq \bar{y}_{c2}$, it follows that:

$$\Delta_2(\bar{y}_{c1}(F_p^1), y_{p1}) \geq \Delta_2(\bar{y}_{c1}(F_p^2), y_{p1}) \Leftrightarrow \bar{y}_{c1}(F_p^1) \geq \bar{y}_{c1}(F_p^2). \blacksquare$$

Proof of Proposition 7

Recall that expected regret is computed according to:

$$\bar{R} = \int_{\gamma_p} \left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_p)} -\Delta_2(y_{c2}, y_{p2}) dF_c \right] dF_p.$$

We begin by showing that $\Delta_2(\cdot, y_{p2})$ is an increasing and concave function in the range $y_{c2} \in (\tilde{y}_{c2}, \bar{y}_{c2})$. Lemma 1 establishes that Δ_2 is increasing in this range. As for concavity,

$$\frac{\partial \Delta_2(y_{c2})}{\partial y_{c2}} = (y_{c2} - \gamma_c)^{-\alpha} - \frac{1}{n} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha}$$

and

$$\frac{\partial^2 \Delta_2(y_{c2})}{\partial y_{c2}^2} = -\alpha (y_{c2} - \gamma_c)^{-\alpha-1} + \frac{\alpha}{n^2} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha-1} < 0,$$

where the inequality follows from the fact that, in this range,

$$y_{c2} - \gamma_c < \frac{y_{c2} + y_{p2} - \gamma_p}{n}.$$

Define:

$$G(x) \equiv \int_{\tilde{y}_c}^x [F_c^1 - F_c^2] dy_{c2},$$

with $G(x) \leq 0$ from second-order stochastic dominance, and $G(\tilde{y}_{c2}) = 0$. Further,

$$dG(x) = F_c^1(x) - F_c^2(x).$$

We have then:

$$\begin{aligned} \bar{R}(F_c^1) - \bar{R}(F_c^2) &= - \int_{\gamma_p} \left[\int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \Delta_2(y_{c2}, y_{p2}) d(F_c^1 - F_c^2) \right] dF_p \\ &= - \int_{\gamma_p} \left(\begin{array}{c} \Delta_2(y_{c2}, y_{p2}) [F_c^1 - F_c^2] \Big|_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \\ - \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{c2}) (F_c^1 - F_c^2) dy_{c2} \end{array} \right) dF_p \\ &= \int_{\gamma_p} \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{c2}) dG(y_{c2}) dy_{c2} dF_p \\ &= \int_{\gamma_p} \left(\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{c2} G(y_{c2}) \Big|_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} - \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} \partial^2 \Delta_2 / \partial y_{c2}^2 G(y_{c2}) \right) dF_p \\ &= \int_{\gamma_p} \left(\begin{array}{c} (\partial \Delta_2(\bar{y}_{c2}(y_{p2}), y_{p2}) / \partial y_{c2}) G(\bar{y}_{c2}) \\ - \int_{\tilde{y}_{c2}(y_{p2})}^{\bar{y}_{c2}(y_{p2})} (\partial^2 \Delta_2 / \partial y_{c2}^2) G(y_{c2}) \end{array} \right) dF_p \leq 0, \end{aligned}$$

where we are using $\Delta_2(\bar{y}_{c2}, y_{p2}) = 0$ and $F_c^1(\tilde{y}_{c2}) = F_c^2(\tilde{y}_{c2}) = 0$.

Then, under $\alpha < 1$, $\bar{y}_{c1}(F_c^1)$ which solves:

$$\Delta_1(\bar{y}_{c1}) = \bar{R}(F_c^1),$$

must be smaller than $\bar{y}_{c1}(F_c^2)$, the solution to

$$\Delta_1(\bar{y}_{c1}) = \bar{R}(F_c^2). \blacksquare$$

Proof of Proposition 8

We begin by showing that $\Delta_2(y_{p2})$ is convex in the range $y_{p2} \in (\hat{y}_{p2}, \tilde{y}_{c2})$. We have:

$$\frac{\partial \Delta_2(y_{p2})}{\partial y_{p2}} = -\frac{1}{n} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha} < 0$$

and

$$\frac{\partial^2 \Delta_2(y_{p2})}{\partial y_{p2}^2} = \frac{\alpha}{n^2} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha-1} > 0.$$

Then, $-\Delta_2(y_{p2})$ is increasing and concave.

Define:

$$G(x) \equiv \int_{\hat{y}_{p2}}^x (F_p^1 - F_p^2) dy_{p2},$$

with $G(\hat{y}_{p2}) = 0$, by assumption, $G(x) \leq 0$, from second-order stochastic dominance, and $dG(x) = F_p^1(x) - F_p^2(x)$.

We have:

$$\begin{aligned} \bar{R}(F_p^1) - \bar{R}(F_p^2) &= - \int_{\gamma_c} \left[\int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} [\Delta_2(y_{c2}, y_{p2})] d(F_p^1 - F_p^2) \right] dF_c \\ &= - \int_{\gamma_c} \left[\int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \Delta_2(y_{c2}, y_{p2}) (F_p^1 - F_p^2) \Big|_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \right. \\ &\quad \left. - \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) (F_p^1 - F_p^2) dy_{p2} \right] dF_c \\ &= \int_{\gamma_c} \left(\int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) (F_p^1 - F_p^2) dy_{p2} \right) dF_c \\ &= \int_{\gamma_c} \left(\int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} (\partial \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}) G(y_{p2}) \Big|_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} \right. \\ &\quad \left. - \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} (\partial^2 \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}^2) G(y_{p2}) dy_{p2} \right) dF_c \\ &= \int_{\gamma_c} \left(\int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} (\partial \Delta_2(y_{c2}, \tilde{y}_{p2}) / \partial y_{p2}) G(\tilde{y}_{p2}) \right. \\ &\quad \left. - \int_{\hat{y}_{p2}(y_{c2})}^{\tilde{y}_{p2}(y_{c2})} (\partial^2 \Delta_2(y_{c2}, y_{p2}) / \partial y_{p2}^2) G(y_{p2}) dy_{p2} \right) dF_c \geq 0. \end{aligned}$$

Therefore, when $\alpha < 1$, $\bar{y}_{c1} (F_p^1) \geq \bar{y}_{c1} (F_p^2)$. ■

Proof of Lemma 9

Since $\bar{R} > 0$, from Proposition 3 we know that $\bar{y}_{c1} > \bar{y}_{c2}$. From Lemma 1, we know that $\partial\Delta_2/\partial y_{c1} > 0$ for $y_{c1} > \bar{y}_{c2}$, when $\alpha < 1$. Further, a pair (\bar{y}_{c1}, y_{p1}) that solves (8) implies that, when holding \bar{y}_{c1} constant, we are in the decreasing range of $\Delta_2(\bar{y}_{c1}, \cdot)$, and y_{p1} is smaller than \bar{y}_{p2} , $\bar{y}_{p2} > \gamma_p$. From Lemma 2, we know that $\Delta_2(\bar{y}_{c1}, \cdot)$ is strictly decreasing in y_{p1} for $y_{p1} \in [\gamma_p, \bar{y}_{p2})$. Therefore, fully differentiating (8), we get:

$$\frac{\partial\Delta_2}{\partial\bar{y}_{c1}}d\bar{y}_{c1} + \frac{\partial\Delta_2}{\partial y_{p1}}dy_{p1} = 0 \Leftrightarrow \frac{d\bar{y}_{c1}}{dy_{p1}} = -\frac{\partial\Delta_2/\partial y_{p1}}{\partial\Delta_2/\partial\bar{y}_{c1}} > 0,$$

and the result follows. ■

The proof of Lemma 10 builds on auxiliary lemmas A.1 and A.2.

Lemma A.1 *For $n^+ > n$, the transfer and moving-out thresholds decrease with family size: $\tilde{y}_{c2}^{n^+} < \tilde{y}_{c2}^n$ and $\bar{y}_{c2}^{n^+} < \bar{y}_{c2}^n$. Further, for $y_{c2} \geq \tilde{y}_{c2}^n$, the differential between the utility under independence and coresidence increases with family size: $\Delta^{n^+}(y_{c2}) > \Delta^n(y_{c2})$.*

Proof of Lemma A.1

Equation (6) directly implies:

$$\tilde{y}_{c2}^{n^+} < \tilde{y}_{c2}^n \text{ and } \bar{y}_{c2}^{n^+} < \bar{y}_{c2}^n,$$

and also that $\tilde{y}_{c2}^{n^+} < \bar{y}_{c2}^{n^+}$ continues to hold. For $y_{c2} \geq \tilde{y}_{c2}^n > \tilde{y}_{c2}^{n^+}$, since in this range no transfers are given by families with either n or n^+ members, consumption and utility in the state of independence are identical in both cases. Consumption at home, however, is strictly lower in the case of a larger family size: $c_{p2}^{n^+} < c_{p2}^n$, and:

$$\Delta_2^{n^+}(y_{c2}) = u(c_{i2}) - u(c_{p2}^{n^+}) > u(c_{i2}) - u(c_{p2}^n) = \Delta_2^n(y_{c2}). \quad \blacksquare$$

For very altruistic parents and $\alpha < 1$, we can rank the magnitudes of $\Delta_2^{n^+}(y_{c2})$ and $\Delta_2^n(y_{c2})$ over their entire domain:

Lemma A.2 *When the parent is fully altruistic ($\lambda = 0.5$) and $\alpha < 1$, $\Delta^{n+}(y_{c2}) > \Delta^n(y_{c2})$ everywhere.*

Proof of Lemma A.2

Given the previous lemma and the continuity of $\Delta_2^n(y_{c2})$ and $\Delta_2^{n+}(y_{c2})$ with respect to y_{c2} , it suffices to prove that

$$\Delta_2^{n+}(y_{c2}) > \Delta_2^n(y_{c2})$$

for $y_{c2} \in [\gamma_c, \tilde{y}_{c2}^n]$.

We consider first the interval $y_{c2} \in [\gamma_c, \tilde{y}_{c2}^{n+}]$. In this range, the excess utility from independence relative to staying home involves positive transfers for either family size. To show our result in this range, it suffices to show that $\partial \Delta_2^n(y_{c2}) / \partial n > 0$.

$$\Delta_2^n(t_2 > 0) = \frac{1}{1-\alpha} \left\{ \left(\frac{y_{p2} + y_{c2} - \gamma_p - \gamma_c}{\Gamma(n-1) + 1} \right)^{1-\alpha} - \left(\frac{y_{p2} + y_{c2} - \gamma_p}{n} \right)^{1-\alpha} \right\}$$

and,

$$\begin{aligned} \frac{\partial \Delta_2^n}{\partial n} &> 0 \\ \iff \left(\frac{y_{p2} + y_{c2} - \gamma_p}{n} \right)^{1-\alpha} \frac{1}{n} - \left(\frac{y_{p2} + y_{c2} - \gamma_p - \gamma_c}{\Gamma(n-1) + 1} \right)^{1-\alpha} \frac{\Gamma}{\Gamma(n-1) + 1} &> 0 \\ \iff (1-\alpha) \left(u(c_{p2}^n) \frac{1}{n} - u(c_{i2}) \frac{\Gamma}{\Gamma(n-1) + 1} \right) &> 0. \end{aligned}$$

For $y_{c2} \in [\gamma_c, \tilde{y}_{c2}^{n+}]$, we know that $u(c_{p2}^n) > u(c_{i2})$. When $\lambda = 0.5$, $\Gamma = 1$, and the result immediately follows. Finally, to show the result for $y_{c2} \in (\tilde{y}_{c2}^{n+}, \tilde{y}_{c2}^n)$, it suffices to show that the slope of $\Delta(y_{c2})$ computed under zero transfers increases with n . Since we have shown that $\Delta_2^{n+}(y_{c2})$ is above $\Delta_2^n(y_{c2})$, at $y_{c2} = \tilde{y}_{c2}^{n+}$, if the slope of $\Delta_2^{n+}(y_{c2})$, which we know is positive, exceeds that of $\Delta_2^n(y_{c2})$ for every $y_{c2} \in (\tilde{y}_{c2}^{n+}, \tilde{y}_{c2}^n)$, then $\Delta_2^{n+}(y_{c2})$ must remain above $\Delta_2^n(y_{c2})$ for these income values.

We have:

$$\Delta_2^n(y_{c2})|_{t=0} = \frac{1}{1-\alpha} \left\{ (y_{c2} - \gamma_c)^{1-\alpha} - \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{1-\alpha} \right\},$$

and

$$\begin{aligned}\frac{\partial}{\partial y_{c2}} [\Delta_2^n (y_{c2}) |_{t=0}] &= (y_{c2} - \gamma_c)^{-\alpha} - \frac{1}{n} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha}, \\ \frac{\partial^2}{\partial y_{c2} \partial n} [\Delta_2^n (y_{c2}) |_{t=0}] &= \frac{1}{n^2} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha} - \alpha \frac{1}{n^2} \left(\frac{y_{c2} + y_{p2} - \gamma_p}{n} \right)^{-\alpha},\end{aligned}$$

and the derivative will be positive iff $\alpha < 1$. Since

$$\frac{\partial \Delta_2^{n+} (y_{c2}) |_{t=0}}{\partial y_{c2}} > \frac{\partial \Delta_2^n (y_{c2}) |_{t=0}}{\partial y_{c2}} > \frac{\partial \Delta_2^n (y_{c2}) |_{t>0}}{\partial y_{c2}},$$

it follows that $\Delta_2^{n+} (y_{c2})$ is steeper than $\Delta_2^n (y_{c2})$ for $y_{c2} \in (\tilde{y}_{c2}^{n+}, \tilde{y}_{c2}^n)$. ■

In (y_{c2}, y_{p2}) space, a larger family size makes the schedules $\tilde{y}_{c2}(y_p)$ and $\bar{y}_{c2}(y_{c2})$ steeper. Further, the schedule $\bar{y}_{c2}^{n+}(y_{p2})$ lies to the left of $\bar{y}_{c2}^n(y_{p2})$. Therefore, given parental income, the child will require a lower income in order to move out if she has a larger family.

Proof of Lemma 10

From lemma A.2, we know that the function $\Delta_2^{n+} (y_{c2})$ is everywhere above $\Delta_2^n (y_{c2})$. We need to establish how \bar{R} changes with family size. Since $\Delta_2^{n+} (y_{c2}) > \Delta_2^n (y_{c2})$,

$$\begin{aligned}\bar{R}^{n+} &= \int_{\gamma_p} \int_{\gamma_c}^{\bar{y}_{c2}^{n+}(y_{p2})} [-\Delta_2^{n+} (y_{c2}, y_{p2})] dF_{p2}^1 dF_{c2} \\ &< \int_{\gamma_p} \int_{\gamma_c}^{\bar{y}_{c2}^n(y_{p2})} [-\Delta_2^n (y_{c2}, y_{p2})] dF_{p2}^1 dF_{c2} = \bar{R}^n,\end{aligned}$$

where the inequality follows since we are integrating only over the range where Δ_2 takes negative values and since $\tilde{y}_2^{n+}(y_p) < \tilde{y}_2^n(y_p)$. Since $R^{n+} < R^n$, the root of the equation:

$$\Delta_2^{n+} (y_{c2}) = \bar{R}^{n+},$$

which is the value of \bar{y}_1^{n+} has to be smaller than the root of

$$\Delta_2^n (y_{c2}) = \bar{R}^n,$$

which is \bar{y}_1^n . ■

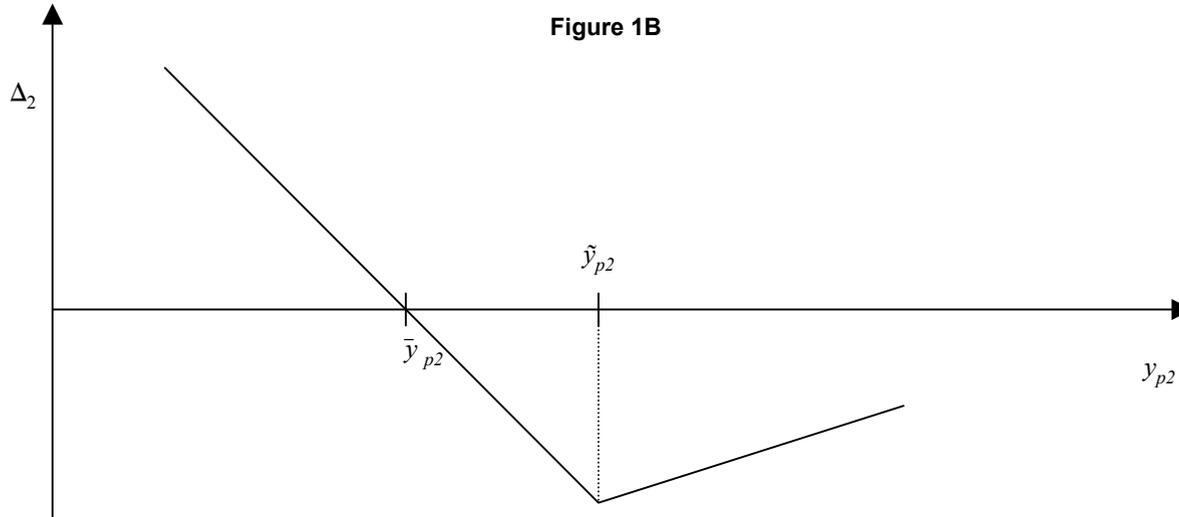
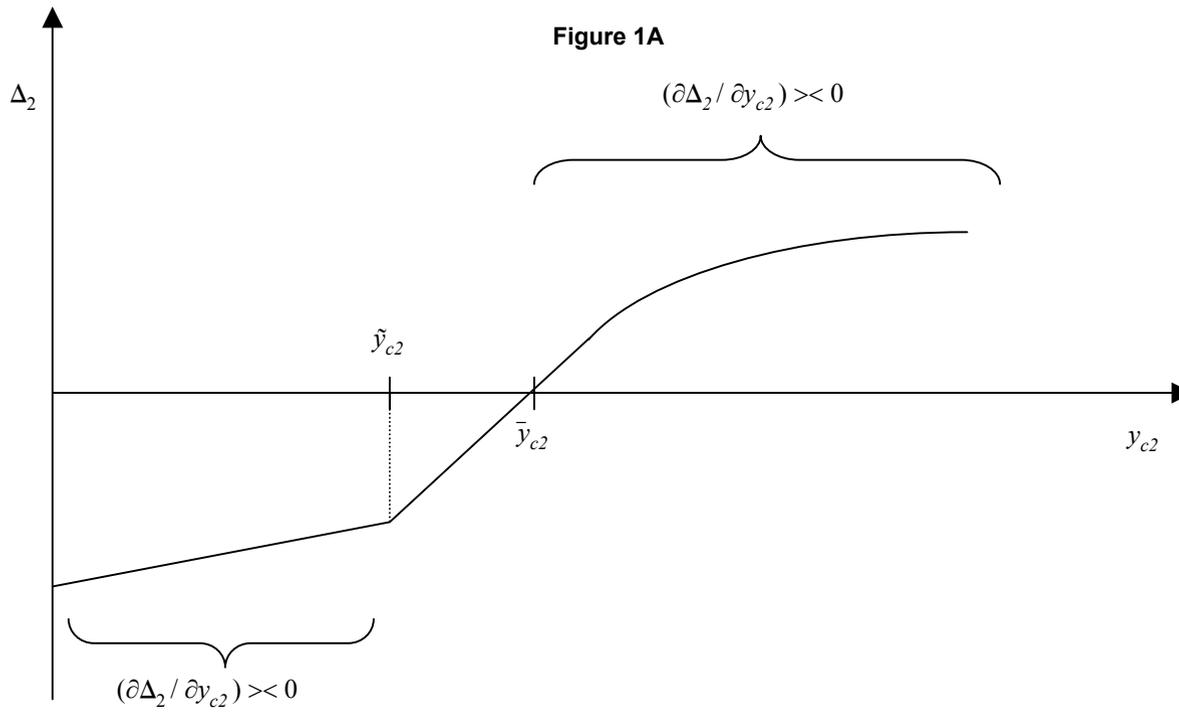


Figure 1. The moving out decision in period 2

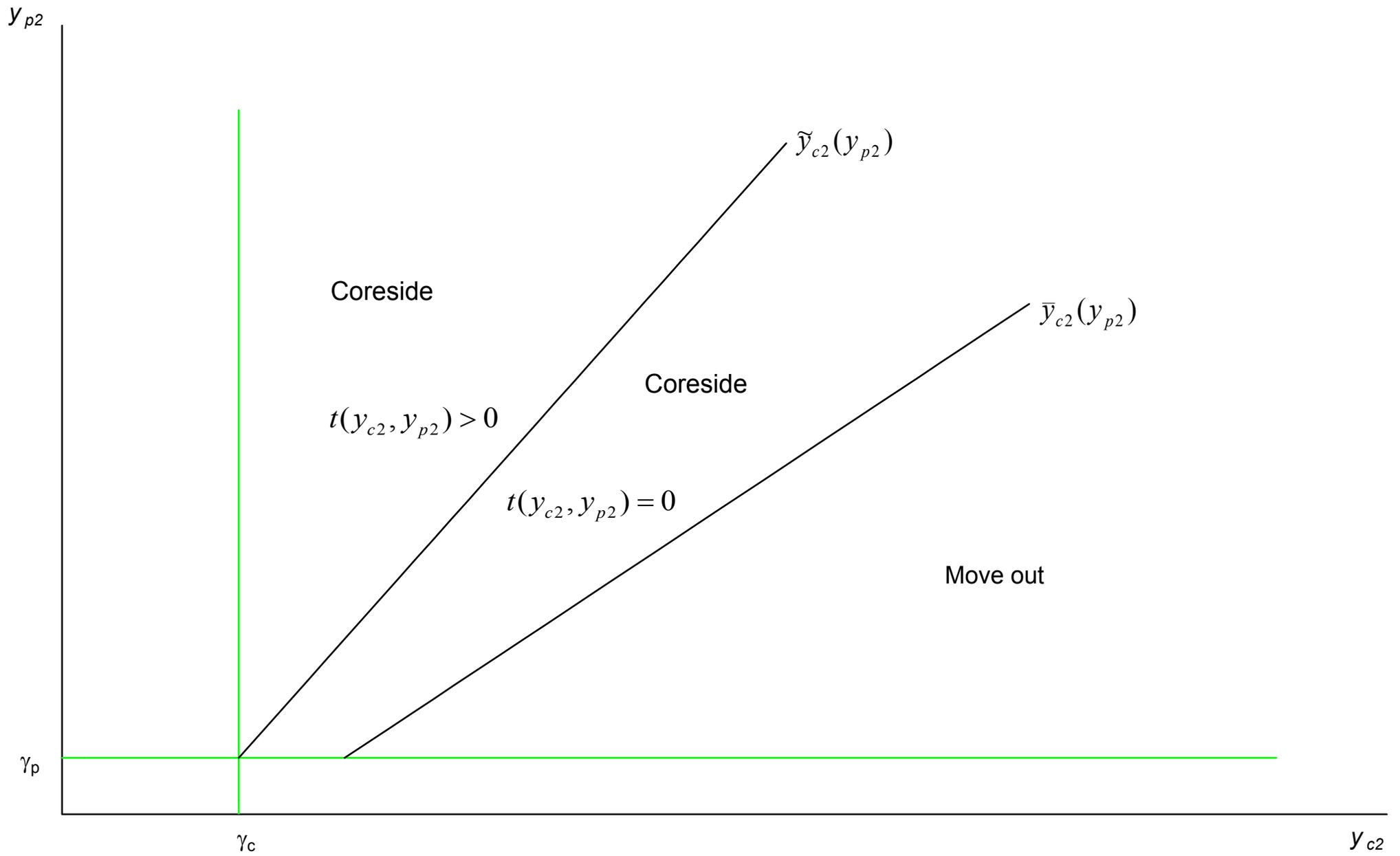


Figure 2. Period 2 residential regimes

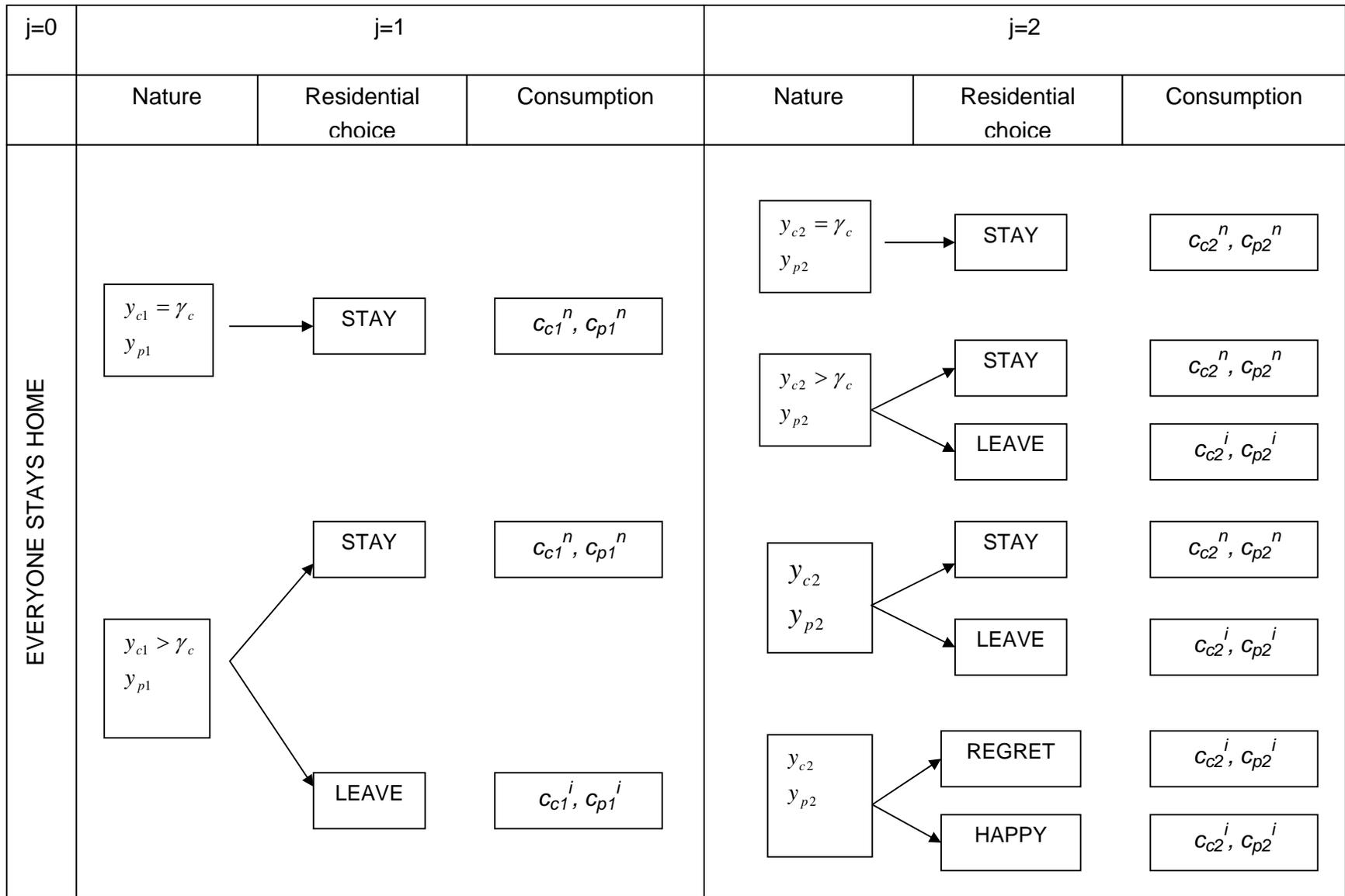


Figure 3. Structure of the model