Altruism, Labor Supply and Redistributive Neutrality

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Abstract

This paper presents a model of familial altruism in which labor supply is chosen endogenously. It is shown that, when the parent helps his child financially, money transfers act as a tax on the child's income and wage. When information asymmetries are added to the model, the parent will offer financial transfers that reward the child relatively more when her income is higher. It is argued that empirical tests of redistributive neutrality are misspecified by not controlling for endogenous labor supply. In addition, to the extent that families operate under information asymmetries, the commonly used panel dataset is not well-suited to test the neutrality null hypothesis.

JEL Codes: D19, D64, D82, J22.

1. Introduction

This paper presents a model of familial altruism in which labor supply is chosen endogenously. The purpose of the analysis is two-fold. First, I characterize the allocation of resources (consumption and leisure) that parental transfers implement. Of particular interest is the relationship between altruism-motivated

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transfers and the different income sources of family members (nonlabor income and wages). Second, the results of the analysis are used to interpret and assess empirical tests of redistributive neutrality from the empirical literature.

Redistributive neutrality, a prevalent theme in the altruism literature, has often been cast in the following terms. Suppose an altruistic parent is faced with a simultaneous one dollar increment in his income and a one dollar reduction in his child's. If he had been providing his child with financial transfers before the income change took place, he would raise the initial transfer by exactly one dollar, completely offsetting the redistribution of income. This prediction of altruistic behavior has been thoroughly tested in the empirical literature (Altonji, Hayashi, and Kotlikoff [3] is the most complete reference on this matter). As will be shown below, endogenous labor supply introduces important qualifications to the theoretical concept of redistributive neutrality as well as to the measurement and testing of neutrality in the data.

I model a family with two members: parent and child. The child has the option of participating in the labor market, whereas the parent's income is exogenous. Although the child is selfish and cares only about her direct utility, the parent is altruistic towards the child and may wish to give her financial transfers. I consider endogenous labor supply in two different dimensions. The simplest setup is one of complete information, where both parent and child observe the non-labor income of both family members as well as the child's wage. More importantly, the child's effort, interpreted in this context as the number of hours worked, is also publicly observed. The problem of resource allocation in the family is framed in the popular Barro [5] and Becker [6] model of altruism here enlarged to allow for the explicit consideration of labor supply. In a static context, the parent chooses how much money to transfer to the child and the child chooses how many hours to work.[1] Transfers and hours worked completely determine consumption of both family members.

An implication of the analysis is the fact that parental transfers act as a tax on the income and wages of both family members, as follows. Since altruistic parents share increments in income between themselves and the child through an adjustment of the financial help they give their offspring, when the child's non-

[1] As will be argued below, conditional on a transfer amount, parent and child would agree on the number of hours that the child should work; it is therefore immaterial whether we consider the parent or the child as the decision makers in what concerns time spent in the labor market. For simplicity, in the analysis, the father is in fact the single decision maker, choosing transfers and hours.

labor income goes up her net income rise after transfers will be smaller than the initial windfall: transfers tax away a fraction of the increment in income. In fact, transfers affect behavior in the very same way as a tax on income would. As an example, the nonlabor income elasticity of the child's labor-supply will be smaller (in absolute value) for a transfer-recipient child relative to that of a kid who does not receive financial help. Similarly, consumption of transfer-giving parents is less income elastic compared to similar individuals for whom transfers are at a corner. Back-of-the-envelope calculations using a familiar parameterization of the utility function indicate that the differences in elasticities across transfer regimes are substantial.

Under complete information, it is straightforward to show that a transfer-giving parent neutralizes redistribution of nonlabor income. That is, if, for example, the tax authority visits our model family and relabels income, raising the parent's nonlabor income by one dollar and reducing the child's nonlabor income by the same amount, the parent will simply raise the initial transfer by one dollar. No other changes take place: consumption and hours worked are identical before and after the taxman pays his visit. However, redistributive neutrality would not apply if the taxman changes wages in a way that leads to a redistribution of labor income. Even though the parental transfer would also adjust to the new wages, the modified price of leisure now optimally leads to a different choice of hours worked. In this case, transfers will not reinstate the original allocation of consumption and leisure.

I also consider a variation of the model where the child's effort is private information. I now assume that, instead of facing a fixed wage rate, here the child faces a distribution of wages. This distribution depends on the child's effort choice, with high effort dominating shirking in the sense of first-order stochastic dominance. Effort is now interpreted as "how hard the child works" as opposed to the number of hours spent in the labor market. In this variant of the model, the parent first announces a transfer function and the child then selects effort with the knowledge of the parent's announcement. When effort is publicly observed, the timing of moves and the stochastic wage distribution do not alter the qualitative results outline above.

When effort is privately observed by the child, the parent will choose the transfer menu in a way to give his child incentives to work hard. As it is later shown, the optimal transfer function displays the common trade-off between insurance and incentives: the parent will reward the child more when the output realization is more likely to have been obtained under high effort. In other words, in order to be able to give the child incentives to work hard, the parent must constrain transfers to vary with income realizations in ways that have to do with the statistical likelihood that high effort was exerted relative to shirking. We now go back to the neutrality question. Suppose that the child has chosen her level of effort and that income has been realized and observed by the parent. Suppose that, at this point, the taxman visits this family and, just as before, he relabels income: takes one dollar from the child and gives it to the parent. Will a transfer-giving parent raise his transfer by one dollar? The answer is a qualified "Yes." Redistribution will be fully neutralized if the parent was providing positive transfers for all income realizations. In fact, the reshuffling of income has not altered the parent's perception of how hard the child works and, as such, incentive-conveying transfers are not to be adjusted beyond the one dollar increment. However, if there were income realizations for which the parent was not providing transfers, redistribution would not be neutral. Not only would we expect the parent not to restore the child's consumption for these income realizations, but the implied modification of the consumption allocation also changes the parent's desire to compensate the child for other income values. When transfers can be at a corner, redistribution has implications similar to a change in initial conditions, to a shift in the distribution of income, for example.

A different and more important question in this context is "How does the optimal transfer evaluated at parent-child income pair (x, y) compare with the transfer provided when income is instead $(x + 1, y - 1)$?" This is a question about the "slope" of the transfer function, how the optimal reward schedule changes around the number y for the child's income, assuming that the parent's income changes symmetrically relative to the child's. As it is shown below, it turns out that the transfer provided under income pair $(x + 1, y - 1)$, labeled $T(x + 1, y - 1)$, does not fully compensate the child for her income loss: $T(x + 1, y - 1) < T(x, y) + 1$. Since the parent incorporates in the transfer payment the different likelihood that high effort was exerted across income observations, he effectively treats the child's income observation y differently (more favorably) than he treats income observation $y - 1$.

Going to the data, now, suppose that one could observe different parent-child pairs, their labor income and hours worked. In the data, the income variability could be thought as originating from both a change in hours, amounting to an endogenous response to wage changes as portrayed in the model under complete information, as well as from pure randomness coming form the non-degenerate distribution of the child's income in the model of private information. The number of hours worked and the intensity of one's effort are two complementary dimensions under which labor supply is endogenous. Both are pertinent in our understanding of the empirical facts concerning neutrality.

Empirical tests of neutrality have been performed by estimating transfer functions and computing the difference in the transfer derivative with respect to the parent's income minus the derivative with respect to the child's income. Under neutrality, this number should equal unity. These tests have routinely aggregated labor and non-labor income components, rather than considering the two separately. But, as the model under complete information shows, redistributive neutrality applies only to the redistribution of exogenous income sources. Failing to control separately for non-labor income and wages amounts to a misspecification of the neutrality tests. The extension of the Barro-Becker model to incorporate labor supply provides a formula relating the relevant parameter for redistributive neutrality to that most commonly estimated in the literature. Preliminary calculations suggest that the difference is numerically important. The direction of the bias, however, does not seem to rescue the neutrality null-hypothesis. In fact, if leisure is a normal good, changes in exogenous income sources will lead the child to work fewer hours, implying a smaller change in her total income than in the non-labor component. Consequently, the optimal reduction in the transfer provided by parents will correspond to a smaller coefficient (in absolute value) multiplying the change in nonlabor income, and to a greater coefficient (in absolute value) multiplying the smaller change in total income. If these forces dominate over other possible biases stemming form the functional form chosen for the transfer function, then the negative coefficient obtained when full income is the chosen regressor should exceed, in absolute value, the negative coefficient obtained when non-labor income is used separately from wages. Simulation results indicate that this is indeed the case.

The most important qualification concerning the interpretation of neutrality tests, as far as labor supply is concerned, is the possibility that families operate under information asymmetries. In fact, as discussed above, the estimation of transfer functions from panel data would simply map out the optimal transfer function, a transfer menu which rewards some income realizations more than others. Under general conditions, we would expect this function to have a slope smaller than unity, corroborating the magnitudes obtained for the neutrality tests in the literature. However, the slope of the transfer function is not informative regarding neutrality. We would need data that could replicate the visit of the taxman, an unexpected tax reform, for example, in order to assess whether or not transfer undo intergenerational redistribution.

The paper proceeds as follows. In section 2, I briefly describe the familiar problem of individual choice over consumption and leisure. This individual impersonates the child of the altruistic parent introduced in the following section. Section 3 characterizes the optimal transfers as well as the consumption and leisure allocation of the altruistic family. Section 4 introduces private information and shows that the transfer menu provided by the parent entails the familiar trade-off between insurance and incentives. Section 5 analyzes results from the empirical literature under the light of the enlarged Barro-Becker model and the private information setup. It additionally provides some numerical illustrations of theoretical results from section 3. Section 6 concludes.

2. The Child's Problem

This section characterizes the standard consumption/leisure choice of an individual. This person will later impersonate the child member of the altruistic family. Independent consideration of the child's problem is pertinent since the economic behavior of the family will encompass two different regimes, depending on whether financial transfers are positive or zero. When transfers are zero, the child's optimal choices are given by the solution to the standard consumption/leisure problem, as specified below.

2.1. The Model

The child enjoys consumption, c_c, and dislikes work, e. Consumption is assumed to be non-negative, $c_c \in \mathbb{R}_+$, whereas total time is normalized to unity, $e \in [0,1]$. Preferences are given by U_c:

$$U_c = u\left(c_c, 1-e\right), \tag{2.1}$$

where $u(\cdot)$ is C^2, strictly increasing and strictly concave with respect to both arguments. The direct utility function $u(\cdot)$ is also assumed to satisfy, for $e \in [0,1]$, $u_1(0, 1-e) = \infty$, $\lim_{c\to\infty} u_1(c, 1-e) = 0$, and for $c \in \mathbb{R}_+$, $u_2(c, 0) = \infty$.

When going into the labor market, the child receives wage w. She also has exogenous income I_c. Her problem is then to

$$\max_{e\in[0,1]} u\left(I_c + we, 1-e\right).$$

The first-order condition is:

$$u_1 (I_c + we, 1 - e)\, w - u_2 (I_c + we, 1 - e) \leq 0. \tag{2.2}$$

Although there is nothing to preclude not working ($e = 0$) as the optimal choice of labor time, I will assume throughout that the solution is interior, so that (2.2) always holds at equality. This makes the formulation of the familial decision problem and accompanying proofs, in the sections to follow, much less cumbersome. For the purpose of analyzing how the family's choices respond to changes in income or wages, there is no loss in generality.

In appendix A, I make assumptions on the function $u(\cdot)$ so as to ensure the optimality of the solution to the first-order condition (3.6), as well as the normality of consumption and leisure. I also assume that labor supply is positively sloped in (e, w) space. I will be explicit below regarding the results for which the assumptions are used.

The following lemmas characterize the child's behavior. They are straightforward implications of the properties of the utility function $u(\cdot)$ and the normality assumptions. The notation $c_{c,j}$ indicates the derivative of the child's optimal consumption function (in effect, a Marshallian demand function) with respect to the jth argument. Notation generalizes in the obvious way.

Lemma 2.1. *The optimal effort choice $e = e(I_c, w)$ is continuously differentiable*[2] *with $e_1 < 0$ and $e_2 > 0$.*

Lemma 2.2. *The optimal consumption choice $c_c(I_c, w)$ is continuously differentiable with $c_{c,1} > 0$ and $c_{c,2} > 0$.*

Expressions for the derivatives of effort and consumption with respect to income and wages are given in the appendix.

3. The Effort-Enlarged Barro-Becker Model

In this section, I extend the benchmark Barro-Becker model of altruism to include the endogenous choice of labor supply.

[2] If effort had been allowed not to be strictly positive, then for some income values and wage rates, the function describing the optimal choice of effort would have a kink. Consequently, it would only be differentiable away from those income and wage pairs. The derivatives presented here can be interpreted as the derivatives of the more general effort function for income and wage rates such that the optimal choice of hours is strictly positive. Since optimal consumption choices inherit the properties of effort, the same remark applies to the statement of the next lemma. This is also true of the results presented in lemmas 3.3 through 3.6.

3.1. The Model

Consider a family formed of an altruistic parent and his child, the child being the individual described in section 2. For simplicity, it is assumed that only the child works. The subscript p indicates parental variables. Let the constant λ take values in $[0.5, 1]$. Given a familial consumption pair (c_p, c_c), and the child's effort e, the parent's total utility U_p is:

$$U_p = \lambda U(c_p) + (1-\lambda)\, u(c_c, 1-e), \tag{3.1}$$

where the direct utility function $U(\cdot)$ is C^2, strictly increasing and strictly concave. Further, it is assumed that $U'(0) = \infty$ and $\lim_{c\to\infty} U'(c) = 0$. The properties of the child's direct utility function $u(\cdot)$ have been stated above.

The parent receives exogenous income $I_p \in \mathbb{R}_+$, whereas the child's total endowment is the sum of the exogenous component I_c and the labor payments we, with $(I_c, w) \in \mathbb{R}^2_+$. Given I_p, I_c and the market wage w, the parent chooses a non-negative amount of resources he may transfer to the child, denoted T, as well as the child's working hours, e. As in section 2, it will be assumed that income and wages are such that optimal effort is interior.

The child's consumption is then:

$$c_c = I_c + we + T, \tag{3.2}$$

while the parent consumes

$$c_p = I_p - T. \tag{3.3}$$

The transfer and working hours solve[3]

$$\max_{T\geq 0, e\in[0,1]} \lambda U(I_p - T) + (1-\lambda)\, u(I_c + we + T, 1-e). \tag{3.4}$$

First-order conditions are:

$$\lambda U'(I_p - T) \leq (1-\lambda)\, u_1(I_c + we + T, 1-e), \tag{3.5}$$

[3]In the spirit of the Barro-Becker tradition, the effort-enlarged model presented here has all the decision making ability centralized in the parent. Since the child is selfish, it would be desirable to allow the child to select effort and to model the interaction between family members as a game. In [14], I model the interaction between parent and child as a non-cooperative static game. It is shown that the unique Nash-equilibrium of that game replicates exactly the optimal parental choices of the current model, characterized in lemmas 3.2 through 3.6. This is so since the parent cares for the child in a non-distortionary way: conditional on a transfer amount, parent and child would agree on the optimal amount of hours the child should work.

which holds at equality whenever transfers are strictly positive, and

$$u_1 \left(I_c + we + T, 1 - e\right) w = u_2 \left(I_c + we + T, 1 - e\right). \tag{3.6}$$

Let $T(I_p, I_c, w)$ and $e(I_p, I_c, w)$ denote solutions to (3.5) and (3.6). Substituting the optimal choices into (3.2) and (3.3), we get the corresponding familial consumption choices, $c_c(I_p, I_c, w)$ and $c_p(I_p, I_c, w)$.

In appendix B, I make assumptions on $U(\cdot)$ and $u(\cdot)$ so as to ensure the optimality of the solution to the first-order conditions (3.5) and (3.6), as well as the normality of c_p, c_c and e. As before, I assume that the labor supply schedule is positively sloped. I will be explicit below about the role played by these assumptions.

3.2. Results

The solution to the parent's problem belongs to one of two different regimes, depending on whether transfers are positive or zero. The two regimes have associated differences in behavior of family members. As an example, parent and child's response to non-labor income changes will be more elastic when no transfers take place.

Before formally stating the results of the optimization problem described in (3.4), consider the following definitions.

Let $\mathbb{T}$ be the set of income and wage values such that parental transfers are strictly positive:

$$\mathbb{T} \equiv \{(I_p, I_c, w) \in \mathbb{R}^3_+ : T(I_p, I_c, w) > 0\}. \tag{3.7}$$

Let $\mathbb{T}^c$ be the complement of $\mathbb{T}$ with respect to $\mathbb{R}^3_+$: $\mathbb{T}^c = \mathbb{R}^3_+ \backslash \mathbb{T}$.

In the lemmas below, the statement "when transfers are positive" is equivalent to restricting the result to triples (I_p, I_c, w) which belong to the set $\mathbb{T}$. Conversely, "when transfers are zero" refers to values of (I_p, I_c, w) in the *interior* of $\mathbb{T}^c$, the complement of $\mathbb{T}$. Transfers will be exactly zero on the frontier of $\mathbb{T}$. This frontier produces a kink in the transfer function, which carries over to the optimal consumption and leisure choices. (See lemma B.1.)

I will first consider the redistributive neutrality experiment. In the spirit of Barro [5], this experiment amounts to an exogenous relabelling of income, in which some quantity δ is taken from one generation's income and added to the income of the other generation, for example by means of governmental intervention. In what follows, I will consider taking one dollar from the child's (non-labor) income and adding it to the parent's. The question asked under this experiment is then "What would the parental transfer be if the parent knew that, when the triple (I_p, I_c, w) is realized, one dollar of non-labor income will be redistributed in the way just described?" Transfers will neutralize income redistribution if the transfer with income redistribution corresponds to an increment of exactly one dollar over the no-redistribution amount.

Let $T(I_p, I_c, w)$ denote the optimal transfer function provided by the parent in the absence of redistribution, and let $\tilde{T}(I_p, I_c, w)$ denote the corresponding schedule when redistribution takes place. Similarly, let $\tilde{e}(I_p, I_c, w)$ and $e(I_p, I_c, w)$ denote the effort choices with and without redistribution, respectively. Notation generalizes for consumption. Transfers are neutral if $\tilde{T}(I_p, I_{c,} w) = T(I_p, I_c, w) + 1$.

The neutrality experiment does *not* necessarily correspond to verifying how the initial transfer menu $T(\cdot)$ responds under two *different* income pairs, (I_p, I_c) and $(I_p + 1, I_c - 1)$. As will be made clear below (see section 4), if the family operates under an asymmetric information environment, for example, the two experiments yield different results. The source of this distinction hinges on the fact that redistribution is an exogenous phenomenon: transfers possibly adjust to it but family members know which income values were initially in place. The evaluation of the initial transfer menu under different income values entails a possibly different scenario, if family members perceive distinct endowment points as different. This will be the case when the child's income depends in a non-deterministic way on her privately observed effort: income draws are informative about the child's diligence.

Lemma 3.1. *For (I_p, I_c, w) triples such that $T(I_p, I_c, w) > 0$, $\tilde{T}(I_p, I_c, w) = T(I_p, I_c, w) + 1$, $\tilde{e}(I_p, I_c, w) = e(I_p, I_c, w)$.*

Lemma 3.1 states that transfers neutralize income redistribution. The proof (in appendix B) follows from verifying that $\tilde{T}$ and $\tilde{e}$ solve the system of first-order conditions of the parent's problem, equations (3.5) and (3.6). Naturally, this implies $\tilde{c}_c = c_c$ and $\tilde{c}_p = c_p$.

More generally, for any non-negative quantity δ which is redistributed in the way just described, let $\tilde{T}^\delta(I_p, I_c, w)$ denote the transfer prevailing after redistribution, and define $\tilde{e}^\delta(\cdot)$ similarly. We then have

$$\tilde{T}^\delta(I_p, I_c, w) = T(I_p, I_c, w) + \delta, \text{ and } e^\delta(I_p, I_c, w) = e(I_p, I_c, w).$$

The following lemma compares the initial transfer schedule under two *different* income pairs, $(I_p + 1, I_c - 1, w)$ and (I_p, I_c, w). The question answered here is

"How does the parental transfer under (I_p, I_c, w) compare with the transfer the parent will provide if, in turn, $(I_p + 1, I_c - 1, w)$ occurs?"

Lemma 3.2. *For (I_p, I_c, w) triples such that $T(I_p, I_c, w) > 0$, $T(I_p + 1, I_c - 1, w) = T(I_p, I_c, w) + 1$, $e(I_p + 1, I_c - 1, w) = e(I_p, I_c, w)$.*

The proof is identical to the one of the previous lemma. This result states that the optimal transfer schedule offsets perturbations of familial income which leave the sum $I_p + I_c$ constant.

From lemmas 3.1 and 3.2, we have that

$$\tilde{T}(I_p, I_c, w) = T(I_p + 1, I_c - 1, w).$$

In fact, in the present environment devoid of information asymmetries, the two experiments (comparing $\tilde{T}(\cdot)$ with $T(\cdot)$ and comparing $T(I_p + 1, I_c - 1, \cdot)$ with $T(I_p, I_c, \cdot)$), yield the same result. This is true, more generally, for any non-negative income amount δ redistributed within the family:

$$\tilde{T}^{\delta}(I_p, I_c, w) = T(I_p + \delta, I_c - \delta, w).$$

Using the fact that

$$T(I_p + \delta, I_c - \delta, w) = T(I_p, I_c, w) + \delta,$$

the result of the comparison of T under different income draws can be summarized, using "local" notation, by:

$$T_1 - T_2 = 1 \text{ and } e_1 = e_2.$$

Given that the experiments described in lemmas 3.1 and 3.2 have identical results, in section 5, I will refer to the result $T_1 - T_2 = 1$ as corresponding to "neutrality."

The results in lemmas 3.1 and 3.2 depend only on the optimality of the solution (T, e) to the system of first-order conditions (3.5) and (3.6). No other assumptions are invoked for this result.

The appendix establishes the differentiability properties of transfers and effort, which carry over to consumption. As before, let $c_{c,j}$ stands for the derivative of $c_c(\cdot)$ with respect to its jth argument. The following properties of the family's optimal choices are formally shown in appendix B.

Lemma 3.3. *When transfers are positive, $T_1 > 0$, $T_2 < 0$ and $T_3 < 0$.*

The signs of the derivatives of optimal transfers are a direct result of the normality and positively sloped labor supply assumptions. Given normality, transfers have to increase when the parent's income goes up and must decrease when the child is wealthier (so that the parent's consumption may also increase). Since the child works harder when the wage is higher, again the parent benefits from this higher income by reducing the initial transfer. The normality assumptions are additionally used to sign the derivatives of effort and consumption with respect to their arguments (I_p, I_c, w) (lemmas 3.4, 3.6 and 3.8), as well as to compare elasticities across transfer regimes (lemmas 3.5, 3.7 and 3.9).

Lemma 3.4. *When transfers are positive, $e_1 < 0$, $e_2 < 0$ and $e_3 > 0$; when transfers are zero, $e_1 = 0$, and $e(I_p, I_c, w)$ coincides with the effort choice characterized in lemma 2.1:*

$$e(I_p, I_c, w) = e(I_c, w).$$

Consider now a family described by the triple (I_p, I_c, w), for which transfers are positive, and let the number k be the corresponding transfer:

$$T(I_p, I_c, w) = k > 0.$$

Consider additionally another family, described by $\left(\tilde{I}_p, \tilde{I}_c, w\right)$, such that $\tilde{I}_c \equiv I_c + k$, and $\tilde{I}_p$ is such that no transfers take place:

$$T\left(\tilde{I}_p, \tilde{I}_c, w\right) = 0.$$

Clearly, since $U(\cdot)$ satisfies Inada conditions, such an $\tilde{I}_p$ exists.

The child of the family with variables (I_p, I_c, w) has the same amount of post-transfer resources as the child whose family is characterized by $\left(\tilde{I}_p, \tilde{I}_c, w\right)$. The labor supply and consumption of these two individuals solve the relevant first-order conditions. The transfer recipient's choices are determined from the parent's problem, equation (3.6), whereas the individual receiving no transfers chooses according to equation (2.2). Comparison of these two equations shows that the two children experience exactly the same labor supply and consumption.

The following lemma (see proof in appendix B) shows that, confronted with a marginal increase in her non-labor income, the transfer recipient child reduces her labor supply by less than the child who is not benefitting from financial help.

Lemma 3.5. *The child's labor supply response to non-labor income changes is more elastic when she is not receiving transfers:*

$$e_2(I_p, I_c, w) > e_1\left(\tilde{I}_c, w\right).$$

The intuition for this result is the "tax" imposed by the transfer-giving parent over the increment in I_c: by reducing the initial transfer, the parent benefits from the child's greater income. The child who is not being helped does not face such an adjustment and, consequently, benefits from a higher net income gain.

I could not sign the difference in the effort derivatives with respect to the wage across transfer regimes. However, for general utility functions, the two expressions should differ.

Lemma 3.6. *When transfers are positive, $c_{c,1} > 0$, $c_{c,2} > 0$ and $c_{c,3} > 0$; when transfers are zero, $c_{c,1} = 0$, and $c_c(I_p, I_c, w)$ coincides with the consumption choice characterized in lemma 2.2:*

$$c_c(I_p, I_c, w) = c_c(I_c, w).$$

Let (I_p, I_c, w) be such that transfers are positive, and let k, $\tilde{I}_c$ and $\tilde{I}_p$ be defined as above. Then:

Lemma 3.7. *The child's consumption is more elastic with respect to non-labor income when she is not receiving transfers:*

$$c_{c,2}(I_p, I_c, w) < c_{c,1}\left(\tilde{I}_c, w\right).$$

The result in lemma 3.7 follows the same reasoning as that of lemma 3.5. The same intuition also underlies lemma 3.9, below.

Lemma 3.8. *When transfers are positive, $c_{p,1} > 0$, $c_{p,2} > 0$ and $c_{p,3} > 0$; when transfers are zero, $c_p = I_p$.*

Let (I_p, I_c, w) be a triple for which transfers are positive and let k be defined as above. Let $\left(\tilde{I}_p, \tilde{I}_c, w\right)$ be such that $\tilde{I}_p \equiv I_p - k$ and $\tilde{I}_c$ be such that transfers are zero:

$$T\left(\tilde{I}_p, \tilde{I}_c, w\right) = 0.$$

$\tilde{I}_c$ clearly exists, although it may be infinity.

The transfer-giving parent has the same post-transfer income as the parent with non-labor income $\tilde{I}_p$, who provides no financial help to his child. The following lemma shows that the consumption of the latter individual is more elastic with respect to his own income, compared to the consumption of the transfer-giving parent.

Lemma 3.9. *Parental consumption is more elastic when the parent is not providing financial transfers:*

$$c_{p,1}\left(I_p, I_c, w\right) < c_{p,1}\left(\tilde{I}_p, \tilde{I}_c, w\right).$$

Lemmas 3.3 through 3.6 have characterized optimal transfers, effort and consumption. The most important features of these variables can be summarized as follows. Family choices fall in one of two regimes, depending on whether transfers are strictly positive or zero. When transfers are zero, the parent simply consumes the totality of his endowment. Given the preference form assumed for $U_p(\cdot)$, he cares for the child in a non-distortionary way. Consequently, he chooses the consumption and effort allocation preferred by the child, given that transfers are zero. This choice is characterized by the results of section 2. An important aspect of the regime of zero transfers is that the behavior of family members is affected only by their individual income and wage. Consequently, parental consumption does not depend on the child's income and wage, neither the child's consumption and effort depend on the parent's income.

When transfers are strictly positive, on the other hand, individual consumption and effort depend on the wage and income of *all* family members. Moreover, the fact that transfers adjust to changes in the environment causes individuals to respond to income and wage changes differently when they receive/give financial help, compared to the no-transfer regime. Some back-of-the-envelope calculations presented in section 5.2 suggest these different responses are non-negligible.

4. Private Information

In this section, I assume that the child's effort is privately observed[4]. The main results are as follows. It is shown that, under moral hazard, redistributive neutrality

[4]This section borrows from my University of Chicago dissertation, [13].

is still preserved provided transfers are not at a corner. Further, the optimal transfer menu provided by the parent displays the familial trade-off between insurance and incentives. Everything else constant, this causes the parent to compensate the child more when the likelihood that her income was generated under high effort is also higher. Finally, it is argued that, while panel data may enable estimation of the transfer menu under asymmetric information, this does not correspond to the redistributive neutrality experiment.

4.1. The model

In the current scenario, parent and child play a sequential game as follows. The parent's income $I_p \in \mathbb{R}_+$ is now assumed to be random and distributed according to probability density function $\mu(\cdot)$, defined over $\mathfrak{B}(\mathbb{R}_+)$[5]. Effort e is understood here as the intensity with which the child works a fixed number of hours. There is randomness in the wage rate she receives, and the distribution of the wage depends on this intensity. Since hours are fixed, there is no real distinction between the wage and her labor income. For notational simplicty, I denote by I_c the child's total income, a random variable whose distribution depends on e.[6] Further, I_c is drawn from probability density function $f(I_c; e)$, defined over $\mathfrak{B}(\mathbb{R}_+)$. For simplicity, it is assumed that effort can take values in $E = \{e_H, e_L\}$, $e_H > e_L$. The density $f(I_c; e_H)$ dominates — in the sense of first-order stochastic dominance — $f(I_c; e_L)$; further, $\mu(\cdot)$ and $f(\cdot)$ are statistically independent.

The timing is as follows. The parent moves first and announces a menu of transfers $T(I_p, I_c)$, which depends on the future realizations of the endowments. The child then selects a privately observed effort level, $e \in E$. Income realization I_p is drawn from $\mu(\cdot)$ whereas I_c is drawn from $f(\cdot, e)$. Both income realizations are publicly observed. Once the income realizations take place, transfers are implemented according to the announced menu, $T(\cdot)$. Transfers translate into

[5] There is a technical reason for why the parent's income is now stochastic. In section 3, the income of parent and child was observed before the parental transfer was given or effort exerted. Comparing the parental transfer for different values of the family's income was a straightforward experiment. In this section, the timing of moves — described below — prescribes the parent announcing a transfer menu of payments which are contingent on the future observations of I_p and I_c. If I_p is drawn from a degenerate distribution, then the experiment of taking one dollar from the child's income and adding it to the parent's is not well-defined. In other words, the multiplier θ of the incentive compatibility constraint (4.4) would be a function of I_p as opposed to a function of its distribution, as it is in the current case.

[6] If there is a non labor component in the child's earnings, as it was the case in section 3, it is assumed that the parent knows how much it totals.

consumption in the obvious way:

$$c_p = I_p - T(I_p, I_c), \tag{4.1}$$

$$c_c = I_c + T(I_p, I_c). \tag{4.2}$$

Momentary utility has the same form as before:

$$U_p = \lambda U(c_p) + (1-\lambda) u(c_c, 1-e),$$

$$U_c = u(c_c, 1-e).$$

Given the timing of moves, the parent takes into account how the promised menu affects the child's choice of effort. Let E_e denote the expectations operator induced by $\mu(\cdot) f(\cdot, e)$. The parent maximizes his expected utility by choice of the child's effort level e and transfer menu $T(I_p, I_c)$, subject to the child being indifferent between exerting e or its complement e^c:

$$\max_{T(\cdot)\geq 0, e\in E} E_e \{\lambda U(c_p) + (1-\lambda) u(c_c, 1-e)\} \tag{4.3}$$

subject to

$$E_e u(c_c, 1-e) \geq E_{\tilde{e}} u(c_c, 1-\tilde{e}_c), \text{ for } e, \tilde{e} \in E, \tag{4.4}$$

as well as (4.1) and (4.2).

Equation (4.4) is the incentive compatibility condition. I assume that this constraint is binding and also that e_H solves the problem stated in (4.3) and (4.4). Let θ denote the strictly positive multiplier associated with (4.4).

The optimal transfer menu $T(I_p, I_c)$ satisfies the following first-order condition:

$$-\lambda U'(c_p) + u_1(c_c, 1-e_H)\left[(1-\lambda) + \theta\left(1 - \frac{u_1(c_c, 1-e_L)}{u_1(c_c, 1-e_H)} \frac{f(I_c; e_L)}{f(I_c; e_H)}\right)\right] \leq 0, \tag{4.5}$$

which holds at equality whenever T is strictly positive.

Define $F(I_c) \equiv f(I_c, e_L)/f(I_c, e_H)$, commonly known as the *likelihood ratio*. Let $U_1(c_c)$ stand for the ratio of marginal utilities from consumption associated with different effort levels, $U_1(c_c) \equiv u_1(c_c, 1-e_L)/u_1(c_c, 1-e_H)$. When transfers are positive, we may now rewrite the first-order condition as:

$$\lambda U'(c_p) = u_1(c_c, 1-e_H)\left[(1-\lambda) + \theta(1 - U_1(c_c) F(I_c))\right]. \tag{4.6}$$

Inspection of the previous equation shows that, holding other things constant, the child will be rewarded when the odds that a particular realization of I_c was obtained under e_H are high. In fact, a low value of $F(I_c)$ indicates that the probability of I_c being drawn from high effort is large relative to $f(I_c, e_L)$. In turn, a low ratio $F(I_c)$ raises the ratio of the parent's marginal utility over the child's.

Regarding the term $U_1(c_c)$, for separable utility functions, $U_1(\cdot)$ is simply a constant (unity). When the child's utility is not separable in consumption and leisure, $U_1(\cdot)$ is a marginal utility correcting factor, which takes into account how the different effort levels affect the child's marginal utility from consumption. For example, if leisure raises the marginal utility from consumption, then $U_1(c_c) > 1$. The mentioned complementarity between consumption and leisure will cause the parent not to compensate the child as much for high effort, since her marginal utility from consumption is lowered by the child's diligent activity. When $U_1(\cdot) > 1$, this effect, therefore, goes in the opposite direction of a low ratio $F(I_c)$.

Consider now the redistribution experiment of taking one dollar from the child's income and adding it to the parent's endowment. As in section 3, the question here is "What would the parent's transfer be if the parent knew that, upon (I_p, I_c) taking place, one dollar would be redistributed within the family?" Denote by $\tilde{T}(I_p, I_c)$ the new transfer menu under redistribution. The following proposition shows that redistribution is neutral if parents provide positive transfers for all income realizations.

Proposition 4.1. *If the density functions $\mu(I_p)$, $f(I_c; e_H)$ and $f(I_c; e_L)$ are such that $T(I_p, I_c) > 0$, for all realizations of (I_p, I_c), then $\tilde{T}(I_p, I_c) = T(I_p, I_c) + 1$.*

The proof is in appendix C. It amounts to rewriting the first-order condition for positive transfers with the values of I_p and I_c suitably modified, and noticing that once T is substituted for $\tilde{T}$, the first-order condition continues to hold. The need to restrict the result to densities such that transfers are always positive can be understood as follows. If that were not the case, income redistribution for realizations (I_p, I_c) such that transfers are zero would not be undone by the parent. In turn, given that redistribution changes the consumption allocation for at least some of the income realizations, this would also change the "cost" for the parent of making the initial transfer function incentive compatible. In other words, the multiplier associated with the the incentive compatibility constraint would also change. In being at least partially effective, redistribution is modifying the intial conditions, it is having an effect comparable to a change in $\mu(\cdot)$. It is worth emphasizing, however, that redistribution does not affect the parent's perception of how hard the child works. That is, the ratio $F(I_c)$, which adjusts parental compensation in order to give the child hard working incentives, remains unchanged under the redistribution experiment. This is the key fact underlying neutrality, provided transfers are positive for all (I_p, I_c) pairs.

A different question one may want to ask concerns the properties of the initial transfer menu, $T(I_p, I_c)$, in the following sense. When comparing two *different* income pairs, (I_p, I_c) and $(I_p + 1, I_c - 1)$, say, is it also the case that the transfer fully offsets the income changes? In fact, it is feasible for the parent to increase the transfer from $T(I_p, I_c)$ to $T(I_p, I_c)+1$. Is this a property of optimal transfers? The answer to this question is "no" and the intuition is as follows. The different income realizations of the child have associated different values of $F(\cdot)$. This causes the parent to perceive I_c and $I_c - 1$ as different, and the insurance/incentives trade-off described above will reward the child relatively more under I_c, if $F(I_c)$ is lower than $F(I_c - 1)$.

By fully differentiating the first-order condition (4.6) and imposing $dI_p = -dI_c$, one obtains the slope of the transfer menu across income pairs (I_p, I_c) such that $I_p + I_c$ is constant. The result is:

$$dT(I_p, I_c) = \left(1 - \frac{u_1(c_c, 1 - e_H)\,\theta U_1(c_c)\,F'(I_c)}{D}\right), \tag{4.7}$$

where D, the denominator in the previous expression, is given by:

$$D = \lambda U''(c_p) + u_{11}(c_c, 1 - e_H)\left[(1 - \lambda) + \theta(1 - U_1(c_c))\,F(I_c)\right]$$
$$-u_1(c_c, 1 - e_H)\,\theta U_1'(c_c)\,F(I_c).$$

As equation (4.7) indicates, the "slope" of the transfer menu will generally deviate from unity, the value which would entail a complete offset of the income perturbations. The sign of the ratio in (4.7) hinges on the signs of $F'(\cdot)$ and $U_1'(\cdot)$. Having $F'(\cdot) < 0$, a condition know in the literature as the *monotone likelihood ratio* property, is a sufficient condition for $f(\cdot; e_H)$ to first-order stochastically dominate $f(\cdot; e_L)$. The derivative of the ratio $U_1(\cdot)$ would be zero if $u(c, e) = \log(c) - e$, for example. Having $F'(\cdot) < 0$ and $U_1'(\cdot) \geq 0$ is sufficient for $dT(I_p, I_c) < 1$.[7] This implies that transfers less than fully compensate the child for income losses, even when the family's total income remains constant.

[7] Note that, from the first-order condition for transfers, equation (4.6), we know that the expression in square brackets in the denominator D is strictly positive when transfers are also strictly positive.

From an algebraic point of view, the transfer slope deviates from unity to the extent that $\theta F'(I_c) \neq 0$. The relevance of the factor $F'(I) \neq 0$ can be understood as follows. Loosely speaking, when the income perturbation takes place, we are comparing two endowment pairs, (I_p, I_c) and $(I_p + 1, I_c - 1)$. The optimal transfer payment, which is constrained to provide incentives for e_H to be exerted, has transfers depend on $F(I_c)$. The derivative $F'(\cdot)$ reflects the need to adjust the transfer payment as a function of the relative likelihood that low effort was exerted. For example, for I_c values such that $F'(I_c) < 0$, the drop in the child's income will not be fully compensated by the parent ($dT(I_p, I_c) < 1$). The reduction in the child's post-transfer income ensures that her incentives for hard work remain effective.

Could θ be zero, so that parent and child preferred the same effort choice? It is definitely a possibility. A binding incentive compatibility constraint also depends on the fact that parent and child disagree over the effort choice. This could happen in the current setup since the child's effort choice affects the parent's expected utility via the probability distribution from which I_c is drawn. The potential disagreement between parent and child over effort choices was absent from section 3 since the child's effort did not affect the child's wage or her non-labor income.

An interesting theoretical point is the specificity of the results obtained under asymmetric information, as far as the timing of moves is concerned. In fact, had we considered different models where the child either moves first or at the same time as the parent, the slope of the parental transfer function would have equaled unity in either case. The reason is that the only best-response for the parent to the child's choice of hours is to equalize the marginal utilities of both family members up to the ratio $(1 - \lambda)/\lambda$, just as in the model of section 3. The private information case is special in that income realizations are informative about the child's effort. But the timing of moves has also to be such that this information can be embedded in parental transfers.

The question — "what is the slope of the transfer menu across pairs of family income with the property that the sum $I_p + I_c$ is constant?" — is quite pertinent in view of the interpretation that one may attribute to estimates of transfer functions from panel data. In fact, the data used to estimate transfer functions will typically consist of observations of (I_p, I_c) for each family in the panel. Once demographic and taste elements have been controlled for, this empirical exercise corresponds to an estimate of the transfer function $T(I_p, I_c)$. As described in section 5, tests of redistributive neutrality have been performed by comparing estimates of $T_1 - T_2$ with the reference value of unity, allegedly implied by the null. As shown above, this procedure does not capture the redistributive neutrality result. Under the light of information asymmetries, it at best provides an estimate of how parental transfers react to information on the child's effort.

In view of the previous discussion, it is not surprising that redistributive neutrality has been overwhelmingly rejected in the empirical literature. A reference value for the difference in the transfer derivatives, obtained by estimating a transfer function using panel data, can be found in Altonji *et al.* [3]. The estimates produced by this study indicate that $T_1 - T_2$ does not exceed 13 cents, a very small magnitude compared to the expected dollar. Other researchers, who also estimate transfer functions (for example McGarry and Schoeni [19], [20]), report — at least heuristically — that the implied difference in transfer derivatives falls far short of the neutrality benchmark. The fact that $T_1 - T_2$ is smaller than one agrees with the results found above, concerning the slope of the transfer function. As proposition 4.1 shows, however, this is not and indication that redistributive neutrality does not hold.

5. Empirical Evidence

In this section, I use the model presented in section 3 to comment on results from the empirical literature. In the absence of information asymmetries, the effort-enlarged Barro-Becker model provides a formula relating the transfer derivative, from the point of view of redistributive neutrality, to the parameter actually estimated[8]. To the extent that families have the ability to adjust their labor supply to changes in income and wages, these two numbers will differ since no adequate control for labor supply has been taken, in the empirical estimates. Back-of-the-envelope calculations of the difference between the appropriate transfer derivative, from the theoretical model, and the simulation analogue of actual empirical estimates, based on a familiar parameterization of the utility function, suggest that the numerical difference between the two parameters is non-negligible. The relative magnitudes of the two simulated parameters additionally indicates that test mis-specification due to not fully controlling for endogenous labor supply should have biased the test *in favor* of the neutrality result.

[8] I am using the fact that the redistributive neutrality experiment and the evaluation of the transfer function of the Barro-Becker model under different income pairs are identical exercises when there are no information asymmetries between family members. (See section 3.)

5.1. Results from the Empirical Literature

A substantial part of recent empirical work on altruism has devoted attention to the properties of financial and time transfers between parents and their adult children. Some examples of this literature include Altonji *et al.* [2], [3], McGarry and Schoeni [19], [20], Cox [9], Cox and Raines [10] and Cox and Rank [11]. Of particular interest, from the point of view of altruism, is the concern about whether or not financial transfers are increasing in the income of the donor and decreasing in the recipient's. Another empirically examined property of transfers, presumed to hold under the null hypothesis of altruism, is redistributive neutrality.

In section 3, optimal transfers from parent to child were characterized and shown to take the form $T(I_p, I_c, w)$. Transfers were also shown to verify redistributive neutrality. One possible empirical approach, in order to estimate transfer functions, would be to specify a functional form for the transfer equation, taking into account how demographic factors such as family size and age composition may affect the propensity and amount of transfers. Generalizing the transfer function to depend on the parent's wage[9], I now use w_p to denote the parent's wage and similarly for the child's. One could then write the following empirical equation:

$$T = \alpha + \beta_1 I_p + \beta_2 I_c + \gamma_1 w_p + \gamma_2 w_c + \delta_p X_p + \delta_c X_c + u, \tag{5.1}$$

where X denotes a vector of demographic variables and u is a random disturbance assumed to be drawn from a known distribution. In this context, T represents the amount of financial transfers from parents to their children, for household i, in period t. The parameters in (5.1) could then be estimated from data on a cross-section of households, provided information was collected on kinship and transfers (in addition to the obvious income and wage data, as well as the demographic variables). The linear functional form in (5.1) is not a limitation since non-linearities can be easily accommodated by including regressors in the powers of the explanatory variables. Since transfers are observed only when positive, equation (5.1) must be estimated non-linearly. Tobit models have been particularly popular in the empirical literature. Theory predicts that $\beta_1 - \beta_2 = 1$ —

[9] The model of section 3 did not consider the choice of parental labor supply. By including the parent's wage in the empirical equation (5.1), I am considering here the more realistic generalization of the model, with parents participating in the labor market and earning wage w_p. I have derived all the results concerning how transfers, time at work and familial consumption vary with income and wages for this model, when utility is separable in consumption and leisure and parent and child have the same momentary utility function. The results are simply generalizations, in the natural way, of those presented in lemmas 3.1 through 3.6, as well as B.1. Interesting extensions include $\partial e_p / \partial w_c < 0$ and $\partial e_c / \partial w_p < 0$, when transfers are positive.

redistributive neutrality — while no particular numerical value is assigned to the difference $\gamma_1 - \gamma_2$.

The properties of transfers have been analyzed using versions of equation (5.1) of the following form:

$$T = a + b_1 I_p^S + b_2 I_c^S + d_1 X_p + d_2 X_c + v, \tag{5.2}$$

where the superscript S indicates total income: the sum of labor and non-labor income. The child's total income, in the notation of section 3, is then $I_c^S = I_c + we$. Thorough empirical experimentation has estimated 5.2 using several different possibilities for the income variables, including current and permanent income. All the references cited above have found evidence that the probability of a transfer being provided depends positively and in a significant way in the donor's income, and depends significantly on the recipient's income, although with a negative coefficient. Concerning amounts given, with the exception of Cox [9], Cox and Raines [10], and Cox and Rank [11][10], actual transfers were found to display the same sign pattern as the probability that one was given.

In all the work cited here, although the topic is only seriously considered in Altonji *et al.* [3], reference has been made to the redistributive neutrality test. Redistributive neutrality has been interpreted as the statement that the difference between the transfer derivatives with respect to parent and child's income should equal unity. Using the notation of the test equation above, this translates into $b_1 - b_2 = 1$. As mentioned above, Altonji *et al.*'s estimates of this difference do not exceed 13 cents.

In section 3, when characterizing the properties of transfers, it was stressed that income redistribution within the family was neutral with respect to resource allocation only when non-labor income redistribution was considered. Under the assumption that parent and child can adjust their working hours in response to changes in wages or exogenous income sources, redistribution of labor-income is not a well-defined experiment, in general. In fact, if parent and child face different wage rates, they will engage in different adjustments of their labor force participation when faced with changes in their labor income. This is so even when utility is separable in consumption and leisure as changes in income or wages still induce adjustments in labor force participation in that particular case.

The Barro-Becker model (section 3), enables us to relate the parameter of interest concerning redistributive neutrality, the coefficient β_2 in equation (5.1),

[10] See Fernandes [13] on estimation procedures — generalized Tobit — and potential problems with the datasets used in [9], [10] and [11].

with the parameter actually estimated, b_2, from equation (5.2). The child's total income relates to labor income as follows:

$$I_c^S = I_c + we.$$

Suppose that the exogenous component of I_c^S is very small, so that most of the changes in I_c^S are due to changes in wages and labor force subsequent adjustment[11]. Then, changes in I_c^S relate to changes in the wage rate w as follows:

$$dI_c^S = \left[1 + \frac{w}{e}\frac{\partial e}{\partial w}\right] edw. \tag{5.3}$$

Let $\eta_{e,w}$ stand for the elasticity of labor supply with respect to changes in wage:

$$\eta_{e,w} = \frac{w}{e}\frac{\partial e}{\partial w}.$$

Then, we may rewrite (5.3) as follows:

$$dw = \frac{dI_c^S}{\left(1 + \eta_{e,w}\right) e}. \tag{5.4}$$

From the model of section 3, desired transfers depend on the wage rate as well as on the exogenous income components, I_p and I_c. Consider the expression for the derivative of parental transfers with respect to the child's wage, derived in equation (B.15)[12]:

$$\frac{\partial T}{\partial w} = \left[\frac{\partial T}{\partial I_c} - \frac{\partial e}{\partial I_c}\frac{u_1}{\lambda U'' e}\right] e.$$

[11] Whether or not I_c is small does not affect the substance of the results presented here, while simplifying the exposition.

[12] The more general expression, derived for the model when the parent also participates in the labor market, is as follows. The parental utility function $U(c_p)$ is now replaced by $u(c_p) + v(1 - e_p)$, where e_p denotes time spent at work by the parent and $u(\cdot)$ and $v(\cdot)$ have the usual properties. The child's utility function $u(c_c, 1 - e_c)$ is specialized to assume separability and equals the functional form of the parent's utility function, just provided. Then:

$$\frac{\partial T_p}{\partial w_c} = \left[\frac{\partial T_p}{\partial I_c} - \frac{1-\lambda}{\lambda}\frac{u_p'' u_c'}{e_c}\frac{\partial e_c}{\partial I_c} - \frac{w_p u_c'}{v_c'' e_c}\frac{\partial e_p}{\partial I_c}\right] e_c,$$

with u_p'' denoting the second derivative of the parent's utility from consumption, and similarly for the other terms. Considering instead the simpler expression provided in (5.5) is consistent with the enlarged Barro-Becker model presented in section 3 and does not qualitatively alter the subsequent discussion.

Using (5.4), we get:

$$\frac{\partial T}{\partial I_c^S} = \left[\frac{\partial T}{\partial I_c} - \frac{\partial e}{\partial I_c}\frac{u_1}{\lambda U'' e}\right]\frac{1}{(1+\eta_{e,w})}. \tag{5.5}$$

Leaving aside the implications of using the functional form in (5.2) to estimate the transfer function in (5.1), one may think of the number given in (5.5) as the expression that was actually estimated[13]. Recall that the redistributive neutrality property applies to the term $\partial T/\partial I_c$. In fact, the model of an altruistic parent and his child predicts $\partial T/\partial I_p - \partial T/\partial I_c = 1$. Using the notation of the test equations, we may rewrite (5.5) as:

$$\frac{\partial T}{\partial I_c^S} = \left[\beta_2 - \frac{\partial e}{\partial I_c}\frac{u_1}{\lambda U'' e}\right]\frac{1}{(1+\eta_{e,w})} \simeq b_2. \tag{5.6}$$

It is worth comparing the actual estimate b_2 with β_2. The term in brackets is more negative than $\partial T/\partial I_c$, from the assumption that leisure is normal. On the other hand, to the extent that the elasticity of labor supply is positive (negative), this reduces (raises) the magnitude of actual estimates. If the effect of the labor supply elasticity dominates, in the sense of outweighing the effect of the second parcel of (5.5), then b_2 will be strictly smaller than β_2, in absolute value. Since neutrality tests have been performed by computing the difference $\partial T/\partial I_p^S - \partial T/\partial I_c^S$, and the estimates of $\partial T/\partial I_c^S$ have been found to be negative, "compressed" estimates of $\partial T/\partial I_p^S$ and $\partial T/\partial I_c^S$ due to the dampening effect of the labor supply wage elasticity could help explain the very low value of the "test" results, which have been found to be significantly below unity. Although estimates of male labor supply wage elasticities tend to be negative (around -0.1), female labor supply elasticities are positive and more elastic (around 0.2)[14]. The number $\eta_{e,w}$ represents the wage elasticity of the child's household. As such, when head and spouse are present, it will not correspond exactly to any of these estimates but instead it reflects their joint hours response to wage changes.

In an attempt to gain a first impression of the magnitudes at stake regarding the difference $\beta_2 - b_2$, I have chosen a familiar parameterization of the utility functions of parent and child. Selecting a specific functional form enables one to solve the parental decision problem described in section 3. I have used PSID data

[13] The actual estimate, without the assumption that I_c is small, would be an weighted average of the coefficient presented in (5.5) and $\partial T_p/\partial I_c$.

[14] See Borjas [8], pp. 68.

to evaluate the relevant terms, as described below. The same parameterization allowed additional computation of the different elasticities associated with the two transfer regimes also described in section 3.

5.2. Some Numbers

In this section, I assume the following parameterization:

$$U_p = \lambda \frac{c_p^\alpha}{\alpha} + (1-\lambda)\left[\frac{c_c^\alpha}{\alpha} + \frac{H^\beta (1-e)^\beta}{\beta}\right],$$

where H, denotes the total time endowment available to the child's household.

Using this functional form, I solve for the optimal choices of transfers, consumption and effort expressed in terms of I_p, I_c and w. I proceed to compute four elasticity values, corresponding to the child's consumption and effort changes with respect to both non-labor income and the child's wage. Let η_{c_c,I_c} denote the elasticity of the child's consumption with respect to her non-labor income, and $\eta_{c_c,w}$ denote the child's consumption elasticity with respect to her wage. Notation is defined similarly regarding effort responses. I also compute the parameter β_2 and use equation (5.6) to calculate the value of b_2.

My strategy is the following. I obtain data on the child's permanent income, transfers given by the parent, and earnings from table A2-1 in Altonji *et al.* [2]. These are PSID sample mean values. I assume that the child's consumption equals the sum of her permanent income plus net transfers from the parent, and use this number whenever a value for c_c is required in the computations. Since there is no information on hours, I assume a value for the total time endowment H and use the first-order condition with respect to hours in the parent's problem of section 3, equation (3.6), to compute e as follows. This equation can be solved to express e as a function of c_c,

$$e = 1 - \left(\frac{H^\beta}{w} c_c^{1-\alpha}\right)^{\frac{1}{1-\beta}}.$$

To compute the wage, I use the data information on earnings and substitute the actual earnings magnitude divided by eH, the total time spent working, for w, in the previous equation. The fraction of time spent working e is finally computed by resorting to a program which solves non-linear equations. Although there is information on the parent's permanent income and earnings (so one could also calculate the sum of parental permanent income minus transfers given to the

Consumption and Labor Supply Elasticities

η	$\alpha = -1$ $\beta = -1$	$\alpha = -1$ $\beta = -1.5$	$\alpha = 0.4$ $\beta = 0.4$
c_c, I_c+	0.006	0.042	0.052
c_c, I_c	0.006	0.067	0.096
r	0.946	0.624	0.537
e, I_c+	-0.117	-0.032	-0.010
e, I_c	-0.123	-0.052	-0.018
r	0.946	0.624	0.537
$c_c, w+$	0.492	0.444	0.517
c_c, w	0.520	0.712	0.963
r	0.946	0.624	0.537
$e, w+$	0.156	0.043	0.213
e, w	-0.389	-0.164	0.130
r	-0.400	-0.263	1.634
β_2	-0.054	-0.376	-0.463
b_2	-0.954	-0.639	-0.618
r	0.056	0.589	0.750

+ sign indicates positive transfer regime
r is ratio: elasticity (with transfers/no transfers)

Table 5.1: α and β combinations ($\lambda = 0.5$)

children and use this number for c_p), I chose to resort again to one first-order condition from the parent's problem and compute c_p as a function of the child's consumption[15]. The value for parental consumption is only used in the tables below to compute the coefficient b_2. The results are not sensitive to the procedure used in computing c_p.

[15]Optimal c_p is related to c_c as follows:

$$c_p = c_c \left(\frac{\lambda}{1-\lambda} \right)^{1/(1-\alpha)}.$$

Consumption and Labor Supply Elasticites

η	$\lambda = 0.5$	$\lambda = 0.65$	$\lambda = 0.80$	$\lambda = 0.95$
c_c, I_c+	0.052	0.028	0.010	0.001
c_c, I_c	0.096	0.096	0.096	0.096
r	0.537	0.292	0.103	0.008
e, I_c+	-0.010	-0.005	-0.002	0.000
e, I_c	-0.018	-0.018	-0.018	-0.018
r	0.537	0.292	0.103	0.008
$c_c, w+$	0.517	0.281	0.099	0.008
c_c, w	0.963	0.963	0.963	0.963
r	0.537	0.292	0.103	0.008
$e, w+$	0.213	0.257	0.291	0.307
e, w	0.130	0.130	0.130	0.130
r	1.634	1.968	2.227	2.356
β_2	-0.463	-0.708	-0.897	-0.992
b_2	-0.618	-1.060	-1.769	-5.444
r	0.750	0.668	0.507	0.182

+ sign indicates positive transfer regime
r is ratio: elasticity (with transfers/no transfers)

Table 5.2: Different λ values ($\alpha = \beta = 0.4$)

I experiment with different values of α, β and H, as well as the altruism coefficient λ. I use as benchmark values $\lambda = 0.5$ and $H = 3120$. The value of λ implies very strong altruism, yet in the absence of a more informed coefficient on caring, it seemed to be a natural starting point for this exercise. Table 5.2 shows how the results vary with λ. The choice of H equals the time in 52 weeks of 40 hours, multiplied by 1.5. This latter number attempts to capture the fact that one household member will typically devote a sizable fraction of his/her time to child rearing or household production activities.

The tables compare the elasticities across transfer regimes. For example, η_{c_c,I_c+} refers to the elasticity of the child's consumption with respect to non-labor income when she is receiving transfers from the parent. When no "plus" sign is present, the numbers refer to the no-transfer regime. The letter "r" indicates the ratio of the transfer over no-transfer magnitudes. In principle, the numbers one could try to compare against reference values in the literature correspond to the no-transfer regime. In fact, we have no independent estimates of how altruism affects the relevant elasticities. One common feature to all tables is the prediction of the model that consumption, say, is more income elastic when transfers are zero. For example, it is always the case that $\eta_{c_c,I_c+} < \eta_{c_c,I_c}$ (and similarly for the absolute value of the effort responses with respect to non-labor income).

With the mentioned choices of λ and H, I tried several combinations of α and β, as shown in table 5.1. As in other parameter experiments below, I attempted to generate realistic values for the wage elasticity of labor supply, the parameter most likely to be relevant for policies involving inter-cohort redistribution. Blundell, Duncan and Meghir [7] provide estimates of female labor supply wage elasticities which range between 0.13 and 0.371 (table IV).

I started with $\alpha = \beta = -1$, since the implied elasticity of intertemporal substitution is 0.5, a value commonly used in the literature[16]. As shown in table 5.1, all the elasticities are virtually identical across transfer regimes, exception being made to the labor supply wage elasticity. The behavior of $\eta_{e,w}$ is, indeed, very different depending on whether transfers are positive or zero. We see that, when no transfers take place, the parameters imply a backward sloping labor supply ($\eta_{e,w} = -0.38$), whereas we have a positive wage elasticity when transfers are positive ($\eta_{e,w+} = 0.16$). The intuition for this difference is the smaller income effect associated with the regime of positive transfers, since parents tax the income windfall of a higher wage by reducing financial help. It is worth pointing out that the value of η_{e,I_c} is very close to estimates in Blundell *et al.* [7]. The difference between b_2 and β_2 is fairly sizable, although the relative magnitudes (b_2 much smaller, in absolute value, compared to b_2) goes against the possibility that neutrality tests have rejected the null by not having properly controlled for labor supply.

Since a negative value of $\eta_{e,w}$ does not accord with reasonable estimates of labor supply wage elasticities, I experimented with other parameter values. One set of parameter values acceptable on this dimension is $\alpha = \beta = 0.4$, delivering $\eta_{e,w} = 0.13$. Looking at the differences between $\eta_{e,w+}$ and $\eta_{e,w}$, labor supply is over one and a half times more elastic in the positive transfer regime. Qualitatively

[16] Given that the model at hand is static, the interpretation of this coefficient is no longer straightforward. I nonetheless presented it since I found it a useful benchmark for comparison with other parameter values.

similar differences characterize the ratio $\eta_{c_c,I_c+}/\eta_{c_c,I_c}$, where the lower income effect associated with the positive transfer regime causes consumption to increase less when one receives financial help from relatives, and these differences are also of sizeable magnitude. The sizeable difference in elasticities across tranfer regimes generalize to the other elasticities presented in the table. Regarding the difference β_2/b_2, the implied coefficients are now closer than under the initial parameter combination (β_2, estimated at -0.46, is about 75% of b_2).

Holding $\alpha = \beta = 0.4$ constant, I experiment modifying H and λ. The results are not very sensitive to changes in H (numbers not shown). Regarding λ, table 5.2, the elasticities respond in the expected direction (lower net income effects for the child), and, consequently, higher value of λ amplify the differences across regimes. Considering $\eta_{e,w+}$, for example, labor becomes a lot more wage elastic as λ goes from 0.5 ($\eta_{e,w+} = 0.213$) up to 0.95 ($\eta_{e,w+} = 0.307$). The ratio β_2/b_2 decreases with λ, going from 0.75 to 0.18, for the same two extreme values of λ.

As stated before, these numbers represent a mere illustrative exercise. In future research, proper measurement of the differences of labor supply and consumption elasticities across transfer regimes will be attempted. The purpose of this exercise is simply to provide a first back-of-the-envelope assessment of how relevant behavioral parameters could possibly be modified by the transfer status of individuals. I read the numerical results presented here as indicating that elasticities appear to vary significantly across transfer regimes. It is worth pointing out that, even if the transfer amount is small in a given period, as transfer evidence seems to indicate (see Altonji *et al.* [2]), the fact that one is receiving transfer modifies the marginal response to all income sources, even those that are outside the family's influence. In other words, from an economic point of view, one becomes a different person, an economic agent with different response to environmental or policy changes, when receiving financial help from relatives.

Finally, the discrepancy between β_2, the parameter associated with redistributive neutrality, and b_2, the parameter presumably measured in the empirical literature, also appears to be non trivial. The relative magnitudes of both do not, however, appear to help explain the gap between the alleged neutrality test and the benchmark value of unity for the difference $\beta_2 - \beta_1$.

As shown in section 4, information asymmetries are likely to modify the slope of the transfer schedule away from the neutrality benchmark. A very interesting line of empirical work has started addressing the impact of information asymmetries on transfer behavior (see [21]). Villanueva's results show that parental transfers roughly triple in response to a one dollar income loss associated with a layoff —

from 11 cents, the ordinary transfer, to 31 cents, with the layoff (see table 14 of [21]). Similar results obtain when he controls for income loss due to disability. From a quantitative point of view, information asymmetries appear to be an important dimension in explaining transfer behavior.

Indirect evidence on private information is additionally provided in Jensen [16]. He analyzes the response of remittances from family members who migrated from rural communities to cities, how these remittances respond to different income sources. He finds that the derivative of transfers with respect to income receipts from public programs exceeds, by orders of magnitude, the transfer derivative when other income sources change[17]. This corroborates the possibility that incentives are an important part of transfer response to income variation.

Another possible strategy to estimate redistributive neutrality, in the context of the effort-enlarged Barro-Becker model, would be to control for effort directly. In fact, from the parent's first-order condition with respect to transfers, equation (3.5), we obtain transfers as a function of I_p, I_c, w *and* the child's effort, e. With effort fixed, transfers display the redistributive neutrality property: the parent would compensate a one dollar reduction in the child's total income matched with a one dollar increment in his own income by raising transfers exactly one dollar. Consequently, tests of redistributive neutrality could be performed by enlarging the test equation (5.2) to include (instruments for) the time spent in the labor market by parent and child, respectively e_p and e. There is an attempt to control for time at work in some of the empirical work cited here. In both papers by McGarry and Schoeni, [19] and [20], the regressors include dummies for the cases in which the child is working full time or when she is not working/missing. For the parent, a dummy is included for data points where the head/spouse is not employed. The authors report that the results reject redistributive neutrality. In Altonji *et al.* [2], dummy variables for hours in unemployment (two categorical measures) are included for parent and child. They also report the rejection of redistributive neutrality.

In Altonji *et al.* [1], redistributive neutrality was tested using consumption data. Under the assumption that altruistic family members pool resources, their marginal utility from consumption would be common across family members. It could, therefore, be estimated as a fixed effect. Altonji *et al.* regress individual consumption of family members on the family's total income, the income of the

[17] The lowest ratio of the transfer derivative in response to a reduction in pension income (presumably publicly observed) over the transfer derivative in response to reductions in total familial income is about 2. However, he provides estimates in which this ratio is as high as 10.

particular individual corresponding to that consumption information, as well as on a vector of demographic values. In order to take the endogenous choice of working hours into account, I would stress the results they present in Table 4, where the wage rates of husband and wife are controlled for. Still, as in all the other estimates they present, the coefficient on non-labor income is positive and significant. This result corroborates the empirical failure of redistributive neutrality.

6. Conclusion

This paper has generalized the Barro-Becker model of altruism to consider endogenous determination of labor supply. A full characterization of the family's choices of consumption and leisure has been provided. A two-tier regime emerged from the analysis, in that family members behave differently depending on whether intra-family financial transfers are positive or zero. In fact, when transfers are positive, individual decisions depend on the income and wages of their relatives, whereas no such dependence occurs for zero transfers. In addition, the fact that transfers adjust to changes in income and wages amounts to a tax imposed by the transfer donor on the income and wages of transfer recipients. This taxation effect was shown to modify the elasticities of both transfer donor and recipient with respect to changes in income and wages, compared to the no-transfer case. A rough numerical exercise suggests that, for a familiar parameterization of the utility function, the differences in elasticities across transfer regimes are not negligible.

The results of the model as well as the preliminary numerical illustration indicate that family composition and transfer status have a potentially important role to play in the adequate evaluation of policy measures. Although the analysis did not address the labor market participation decision, the incentives problems associated with the "family transfer tax" extend immediately to this dimension of labor supply. The impact of welfare programs as well as that of unemployment subsidies would seem to hinge crucially on the magnitudes of this tax. To the best of my knowledge, however, there has been no empirical attempt to measure how relevant policy elasticities vary with intra-family transfer regimes.

Another set of issues addressed here has been the theme of redistributive neutrality. In the literature, this term has been stated to imply that transfer giving parents will offset income redistribution across family members by suitably correcting their initial transfer. I have shown that the endogenous choice of labor supply qualifies the neutrality result in that neutrality applies to non-labor income sources only. The effort-enlarged Barro-Becker model was shown to relate the relevant parameter concerning neutrality with the magnitude emerging from empirical estimates. Preliminary calculations suggest the difference is quantitatively significant.

I have also pointed out that the estimation of transfer functions from panel data may not be suitable to test redistributive neutrality if the data is characterized by families optimizing under an asymmetric information setting. The need to convey incentives associated with asymmetric information implies that individuals will be rewarded when their income, say, indicates diligent activity. Consequently, estimates of transfer functions will reflect the underlying incentives strategy which compensates some income draws more than others. The neutrality experiment, however, involves an income relabelling experiment, in which one dollar is taken from the child, say, and given to the parent, a relabelling which does not affect the parent's inference of how hard the child worked. Such an experiment, as shown above, is not captured by the typical panel dataset.

The results presented here leave open many questions. An important current research concern is to quantify the decision tax imposed by transfer givers. If this effect proves to be empirically significant, a broad empirical agenda may be in order in view of adequate policy evaluation and model calibration. Finally, there may be a need to reassess neutrality, both in terms of formulating adequate data environments from which it can be properly measured as well as by using adequate income aggregates in tests of this hypothesis.

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A. The Child's Problem

I characterize here the child's choice of effort, when she acts as a single decision maker, not receiving transfers from the parent. The properties of the implied consumption values are also presented. Recall that corner solutions ($e = 0$) have been ruled out, for simplicity.

Concerning the child's utility function, I make the following assumptions:

Assumption 1 For all $(I_p, w) \in \mathbb{R}^2_{++}$ and all effort levels $e \in [0, 1)$, i) $u_{11}w^2 - 2u_{12}w + u_{22} < 0$, ii) $u_{21} - u_{11}w > 0$, iii) $u_{12}w - u_{22} > 0$.

Part i) of assumption 1 ensures that the child's second-order conditions are satisfied (condition i) is sufficient for a maximum). Assumption 1, parts i) and ii) ensure that leisure is a normal good, while i) and iii) deliver the normality of consumption.

Assumption 2 For all $(I_p, w) \in \mathbb{R}^2_{++}$ and all effort levels $e \in [0, 1)$, $(u_{21} - u_{11}w)\, e - u_1 \leq 0$.

Assumption 2 ensures that work effort varies positively with the wage. In turn, this guarantees that consumption is also higher when the wage is higher.

The previous assumptions are, in a sense, too restrictive, since they have been imposed as global conditions. In fact, assuming that leisure and consumption are normal goods, for example, should be a local condition, holding "close" to optimal choices of effort. In the statement of assumptions 1 and 2, however, it has been assumed that the required local conditions also hold for any possibly non-optimal choice of effort. I have chosen this restrictive conditions for simplicity. The global form of assumptions 1 and 2 will only be used to ensure that the solutions to the optimization problems in sections 2 and 3 are unique. For all other results, "local" statements of these assumptions suffice.

Fully differentiating the child's first-order condition, we get:

$$\frac{\partial e}{\partial I_c} = \frac{u_{21} - u_{11}w}{u_{11}w^2 - 2u_{12}w + u_{22}} < 0, \tag{A.1}$$

where the inequality follows from assumption 1, parts i) and ii). As for changes in the wage rate,

$$\frac{\partial e}{\partial w} = \frac{(u_{21} - u_{11}w)\, e - u_1}{u_{11}w^2 - 2u_{12}w + u_{22}} > 0, \tag{A.2}$$

the inequality following from assumption 1, part i), and assumption 2.

Given the properties of $u(\cdot)$, the optimal effort choice $e(I_c, w)$ is a continuously differentiable function of all arguments. If corner solutions had been allowed, then for income and wage rates (I_c, w) such that the first-order condition (2.2) holds at equality with exactly zero hours of work, the optimal choice function $e(I_c, w)$ would have a kink. Consequently, it would only be differentiable away from those income and wage pairs.

The child's consumption is derived from her resource constraint:

$$c_c(I_c, w) = I_c + we(I_c, w).$$

We have:

$$\frac{\partial c_c}{\partial I_c} = -\frac{u_{12}w - u_{22}}{u_{11}w^2 - 2u_{12}w + u_{22}} > 0,$$

where the inequality follows from assumption 1, parts i) and ii). Changes in the wage rate affect consumption as follows:

$$\frac{\partial c_c}{\partial w} = \frac{e\left(u_{22} - u_{12}w\right) - u_1 w}{u_{11}w^2 - 2u_{12}w + u_{22}} > 0,$$

the inequality following from assumption 1, parts i) and iii). Optimal consumption $c_c\left(I_c, w\right)$ is also a continuously differentiable function of all arguments. If $e = 0$ had been allowed, this function would have kinks for the $\left(I_c, w\right)$ pairs described above.

B. The Effort-Enlarged Barro-Becker Model

Let A denote the Hessian matrix associated with the parental problem. I make the following assumptions:

Assumption 3 For all $\left(I_p, I_c, w\right) \in \mathbb{R}^3_{++}$, all transfers $T \in \mathbb{R}$, $T < I_p$, and all effort levels $e \in [0, 1)$, $u_{11}u_{22} - u_{12}^2 > 0$.

Assumption 3 serves two purposes. Together with assumption 1, part i), it ensures that $|A| > 0$. From the properties of $u\left(\cdot\right)$ and $U\left(\cdot\right)$, we know that the system of first-order conditions (3.5) and (3.6) has a solution. $|A|$ being positive implies that this solution is a maximizer. From the fact that these assumptions hold globally (for all endowment and wage rate values as well as effort choices), it follows that the solution is also unique. Assumption 3 additionally ensures that parental consumption is a normal good.

Assumption 4 For all $\left(I_p, I_c, w\right) \in \mathbb{R}^3_{++}$, all transfers $T \in \mathbb{R}$, $T < I_p$, and all effort levels $e \in [0, 1)$, $-u_1\left(\left(1 - \lambda\right)u_{11} + \lambda U''\right) - \lambda U'' e\left(u_{11}w - u_{21}\right) > 0$.

Assumption 4 ensures that the parent will choose longer working hours for the child when her wage goes up.

Assumption 5 For all $\left(I_p, I_c, w\right) \in \mathbb{R}^3_{++}$, all transfers $T \in \mathbb{R}$, $T < I_p$, and all effort levels $e \in [0, 1)$, $\lambda U''\left[\left(u_{22} - u_{12}w\right)e - u_1 w\right] - u_1 u_{12}\left(1 - \lambda\right) > 0$.

Assumption 5 ensures that the child's consumption goes up with her wage.

The comment made in appendix A concerning the global and, therefore, restrictive nature of the assumptions stated there applies to assumptions 3 through 5, as well.

B.1. The Hessian Matrix

Here, I derive the Hessian matrix of the Barro-Becker model presented in section 3.

Let $A = \{a_{ij}\}_{i,j=1,2}$. A is as follows:

$$A = \begin{bmatrix} (1-\lambda) u_{11} + \lambda U'' & (1-\lambda)(u_{11}w - u_{12}) \\ u_{11}w - u_{12} & u_{11}w^2 + u_{22} - 2u_{12}w \end{bmatrix}.$$

From the properties of the direct utility functions $u(\cdot)$ and $U(\cdot)$, $a_{11} < 0$. After some rearranging, the determinant of A is

$$|A| = \lambda U'' \left(u_{11}w^2 + u_{22} - 2u_{12}w\right) + (1-\lambda)\left(u_{11}u_{22} - u_{12}^2\right).$$

B.2. Proof of Lemmas

Proof of lemma 3.1

The transfer schedule $\tilde{T}(I_p, I_c, w)$ and the corresponding effort $\tilde{e}_c(I_p, I_c, w)$ solve:

$$\lambda U'\left(I_p + 1 - \tilde{T}\right) \leq (1-\lambda) u_1 \left(I_c - 1 + w\tilde{e}_c + \tilde{T}, 1 - \tilde{e}_c\right) \tag{B.1}$$

$$u_1 \left(I_c - 1 + w\tilde{e}_c + \tilde{T}, 1 - \tilde{e}_c\right) w = u_2 \left(I_c - 1 + w\tilde{e}_c + \tilde{T}, 1 - \tilde{e}_c\right). \tag{B.2}$$

Clearly, $\tilde{T} = T + 1$ and $\tilde{e}_c = e$ is a solution to the previous system of equations, with (B.1) holding at equality. Assumption 3 ensures that the solution is unique. In turn, this implies $\tilde{c}_p = c_p$ and $\tilde{c}_c = c_c$. ■

Let $\left(\mathbb{R}_+^3, \mathbb{B}, \mu\right)$ be a measure space, where $\mathbb{B}$ denote the Borel-sets in $\mathbb{R}_+^3$ and μ is the Lebesgue measure.

Lemma B.1. *The transfer and effort choices which solve (3.4), respectively* $T(I_p, I_c, w)$ *and* $e(I_p, I_c, w)$, *and the corresponding consumption of parent and child,* $c_p(I_p, I_c, w)$ *and* $c_c(I_p, I_c, w)$, *are continuously differentiable* μ *- almost everywhere.*

Lemma B.1 follows directly from the properties of $U(\cdot)$ and $u(\cdot)$. The kinks occur for (I_p, I_c, w) triples such that the first-order conditions (3.5) and (3.6) hold at equality when transfers are exactly zero.

Proof of lemmas 3.3 through 3.6.

In the text, it was stated that only $e > 0$ would be considered, for simplicity. Transfer and effort properties, in the possitive transfer regime, follow from fully differentiating the system of first-order conditions with respect to I_p, I_c and w. We get:

$$\frac{\partial T}{\partial I_p} = \frac{u_{11}w^2 + u_{22} - 2u_{12}w}{|A|}\lambda U'' > 0, \tag{B.3}$$

where the inequality follows from assumption 1, part i), and assumption 3.

$$\frac{\partial T}{\partial I_c} = -\frac{(1-\lambda)}{|A|}\left(u_{22}u_{11} - u_{12}^2\right) < 0, \tag{B.4}$$

the inequality following from assumptions 3 and 5.

$$\frac{\partial T}{\partial w} = \frac{1-\lambda}{|A|}\left[-e\left(u_{11}u_{22} - u_{12}^2\right) + u_1\left(u_{11}w - u_{12}\right)\right] < 0, \tag{B.5}$$

the inequality following from assumption 1, part ii), and assumption 3.

Simple algebra shows that $T_1 - T_2 = 1$:

$$\frac{u_{11}w^2 + u_{22} - 2u_{12}w}{|A|}\lambda U'' + \frac{(1-\lambda)}{|A|}\left(u_{22}u_{11} - u_{12}^2\right) = \frac{|A|}{|A|} = 1.$$

Concerning the properties of the optimal effort choices, we have:

$$\frac{\partial e}{\partial I_p} = \frac{(u_{12} - u_{11}w)}{|A|}\lambda U'' < 0, \tag{B.6}$$

from assumption 1, part ii), and assumption 3.

$$\frac{\partial e}{\partial I_c} = \frac{(u_{12} - u_{11}w)}{|A|}\lambda U'' = \frac{\partial e}{\partial I_p}. \tag{B.7}$$

$$\frac{\partial e}{\partial w} = -\frac{1}{|A|}\left[u_1\left((1-\lambda)u_{11} + \lambda U''\right) + \lambda U'' e\left(u_{11}w - u_{21}\right)\right] > 0, \tag{B.8}$$

from assumptions 3 and 4.

Parental consumption is a normal good in terms of I_p and I_c:

$$\frac{\partial c_p}{\partial I_p} = 1 - \frac{\partial T}{\partial I_p} = -\frac{\partial T}{\partial I_c} > 0, \tag{B.9}$$

where I have used $T_1 - T_2 = 1$ and the result in (B.4), above.

$$\frac{\partial c_p}{\partial I_c} = -\frac{\partial T}{\partial I_c} > 0. \tag{B.10}$$

Concerning the effects on parental consumption of changes in the wage:

$$\frac{\partial c_p}{\partial w} = -\frac{\partial T}{\partial w} > 0. \tag{B.11}$$

The child's consumption is also a normal good with respect to exogenous income:

$$\frac{\partial c_c}{\partial I_p} = \frac{\partial T}{\partial I_p} + w\frac{\partial e}{\partial I_p} = \frac{(u_{22} - u_{12}w)}{|A|}\lambda U'' > 0, \tag{B.12}$$

where the inequality follows from assumption 1, part iii), and assumption 3.

$$\frac{\partial c_c}{\partial I_c} = 1 + \frac{\partial T}{\partial I_c} + w\frac{\partial e}{\partial I_c} = \frac{\partial T}{\partial I_p} + w\frac{\partial e}{\partial I_p} > 0, \tag{B.13}$$

where I have used the fact that $T_1 - T_2 = 1$ and $\partial e/\partial I_p = \partial e/\partial I_c$.

Finally, to see how the child's consumption responds to the wage rate:

$$\begin{aligned} \frac{\partial c_c}{\partial w} &= \frac{\partial T}{\partial w} + e + w\frac{\partial e}{\partial w} \\ &= \frac{\lambda U'' \left[(u_{22} - u_{12}w)\, e - u_1 w\right] - u_1 u_{12} (1 - \lambda)}{|A|} > 0, \end{aligned} \tag{B.14}$$

the inequality following from assumption 5.

We may use (B.4) and (B.7) to rewrite (B.5) as follows:

$$\frac{\partial T}{\partial w} = \left[\frac{\partial T}{\partial I_c} - \frac{\partial e}{\partial I_c}\frac{u_1}{\lambda U'' e}\right] e. \tag{B.15}$$

When transfers are zero, the first-order condition with respect to effort, equation (3.6), is the same as the first-order condition in the child's problem, equation (2.2). Therefore, for triples (I_p, I_c, w) such that transfers are zero, $e\,(I_c, w) = e\,(I_p, I_c, w)$, and $\partial e\,(I_p, I_c, w)\,/\partial I_p = 0$. Similarly, $c_c\,(I_c, w) = c_c\,(I_p, I_c, w)$ and $\partial c_c/\partial I_p = 0$.

In this no-transfer regime, we also have $c_p\,(I_p, I_c, w) = I_p$, and $\partial c_p/\partial I_c = \partial c_p/\partial w = 0$.

Proof of lemma 3.5.

The response of the transfer recipient child to changes in her non-labor income, denoted here by $\partial e^{+}/\partial I_c$, is as follows:

$$\frac{\partial e^{+}}{\partial I_c} = \frac{(u_{12} - u_{11}w)}{(u_{11}w^2 + u_{22} - 2u_{12}w) + \frac{(1-\lambda)}{\lambda U''}(u_{11}u_{22} - u_{12}^2)}. \tag{B.16}$$

Let $\partial e/\partial I_c$ denote the labor supply response of the child receiving no transfers:

$$\frac{\partial e}{\partial I_c} = \frac{u_{21} - u_{11}w}{u_{11}w^2 - 2u_{12}w + u_{22}}. \tag{B.17}$$

Since both children have the same amount of post-transfer resources, as argued in the text, the arguments of all expressions involving derivatives of the child's utility function $u(\cdot)$ are the same. Hence, direct comparison of equations (B.16) and (B.17), together with assumption 1, part i) , and assumption 3, delivers the result. ■

Lemmas 3.7 and 3.9 are proved similarly.

C. Private Information

Proof of proposition 4.1.

The first-order condition for positive transfers, when (I_p, I_c) occur, is given in equation (4.5), reproduced below:

$$\lambda U'(c_p) = (1-\lambda)\, u_1(c_c, 1-e_H)\left[(1-\lambda) + \theta\left(1 - U_1(c_c)\, F(I_c)\right)\right].$$

Under the redistribution experiment, the first-order condition for positive transfers is now:

$$\lambda U'\left(I_p + 1 - \tilde{T}(I_p, I_c)\right) = (1-\lambda)\, u_1\left(I_c - 1 + \tilde{T}(I_p, I_c), 1 - e_H\right)$$
$$\left[(1-\lambda) + \theta\left(1 - \frac{u_1\left(I_c - 1 + \tilde{T}(I_p, I_c), 1 - e_L\right)}{u_1\left(I_c - 1 + \tilde{T}(I_p, I_c), 1 - e_H\right)} \frac{f(I_c, e_L)}{f(I_c, e_H)}\right)\right].$$

The two expressions are equivalent when

$$\tilde{T}(I_p, I_c) = T(I_p, I_c) + 1.$$

■